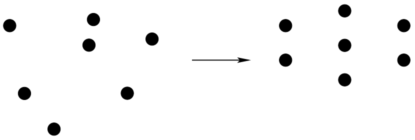
Deterministic Pattern Formation in Swarms of Robots

Franck Petit

Problem



Definition (Arbitrary Pattern Formation)

Given a swarm of n robots scattered on the plan, designing a deterministic algorithm so that, the robots eventually form a pattern $\mathcal{P}$ made of n positions and known by each of them in advance.

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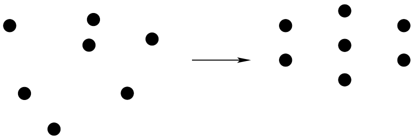

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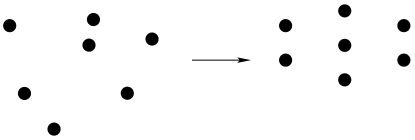

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In other words, at the end of the computation, the positions of the robots coincide, with the positions of $\mathcal{P}$, where $\mathcal{P}$ can be translated, rotated, and scaled in each local coordinate system.

Previous Works

SoD and Chirality

[Flocchini et al., 1999]

A solution exists for any n and any $\mathcal{P}$.

SoD and No Chirality

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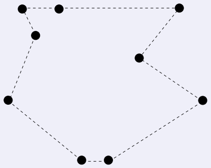

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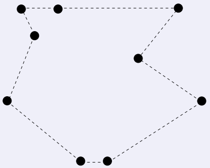

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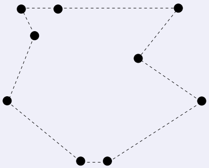

SoD and No Chirality

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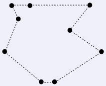
If n is odd, a solution exists for any $\mathcal{P}$.



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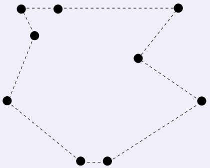

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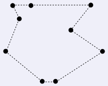
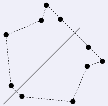


If n is even, a solution exists provided that $\mathcal{P}$ is symmetric.



Pattern Formation With No Sense of Direction

Question

Assuming no sense of direction (with or without chirality), which kind of patterns can be formed in a deterministic way?

Pattern Formation With No Sense of Direction

Theorem

[Flocchini et al., 2001]
If the robots share no Sense of Direction, then they cannot solve the Arbitrary Pattern Formation problem deterministically, even having the ability of chirality.

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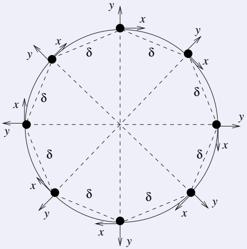

Pattern Formation With No Sense of Direction

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Let Θ be the class of patterns that can be solved deterministically assuming robots devoid of sense of direction.

Corollary

Either Θ *is equal to the set of regular polygons (n-gons)*
or $\Theta = \emptyset$.

Circle Formation

Question

Assuming no sense of direction, is it possible to eventually form a regular n-gon (circle)?

Circle Formation

Previous work

[Défago and Konagaya, 2002] [Chatzigiannakis et al., 2004] [Défago and Souissi, 2008]

Asymptotic convergence toward the n-gon.

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Any n (in ASYNC), but the n-gon is not systematically achieved.

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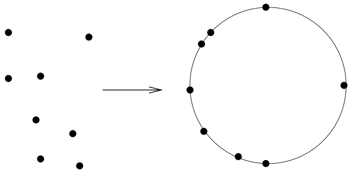

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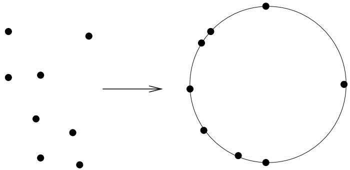

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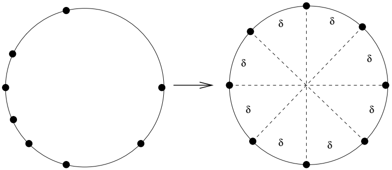

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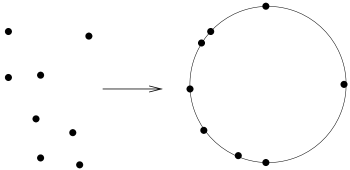

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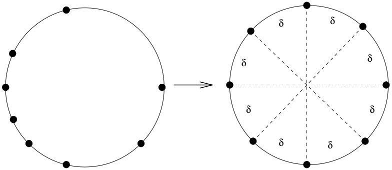

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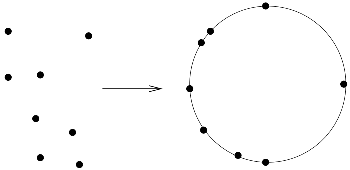

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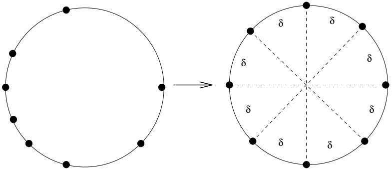

HARD PART

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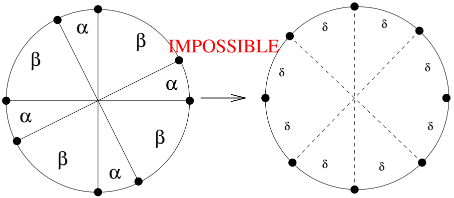

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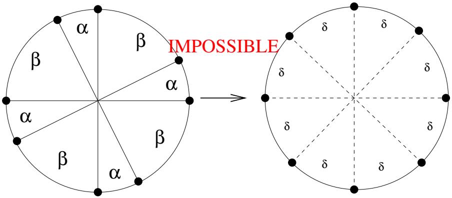

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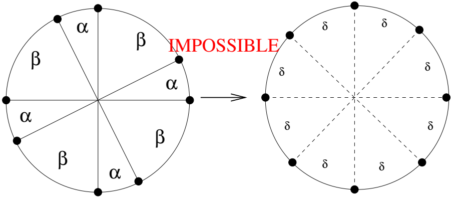

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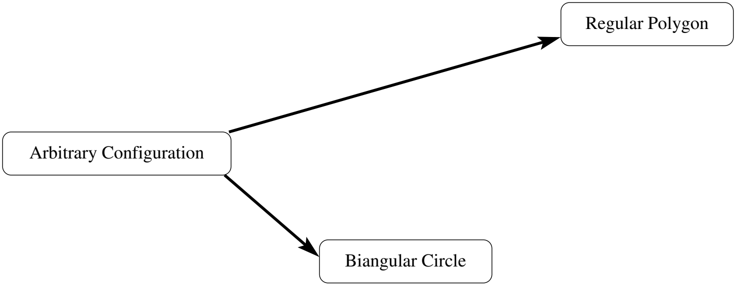
Fortunately, not!

An almost *n*-gon algorithm



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Starting from a biangular circle

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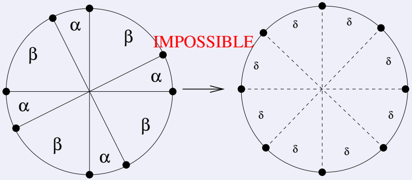

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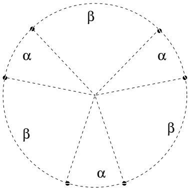

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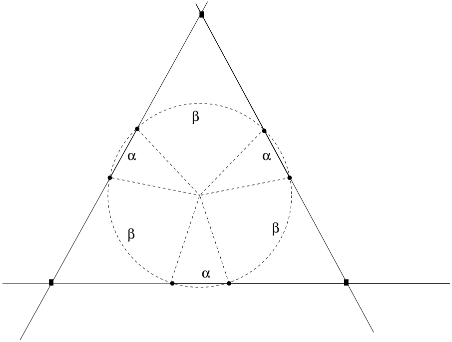

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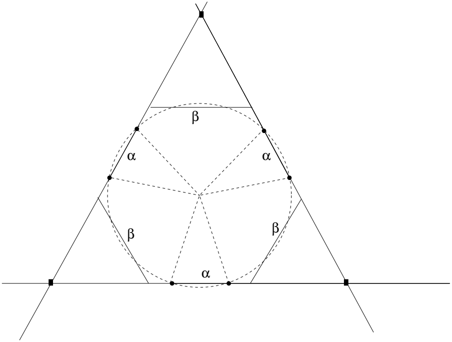

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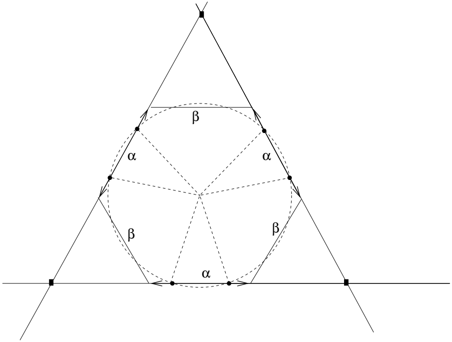

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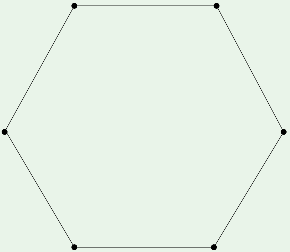
Every robot is active



Regular Polygon

Starting from a biangular circle

Let us leave $\mathcal{C}$

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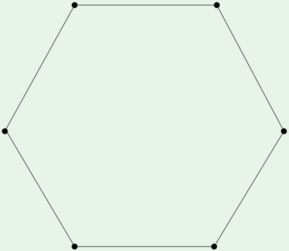

Regular Polygon

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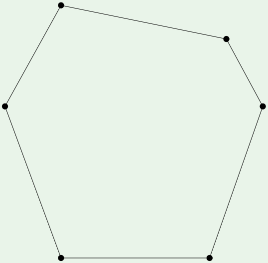

Ideal Polygon

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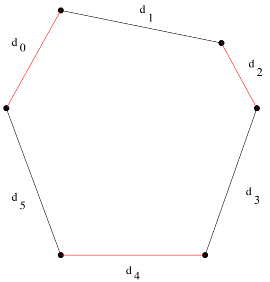

Property 1

- Either $d_0 \geq d_1 \leq \cdots \geq d_{n-1} \leq d_0$
- or $d_0 \leq d_1 \geq \cdots \leq d_{n-1} \geq d_0$

Starting from a biangular circle

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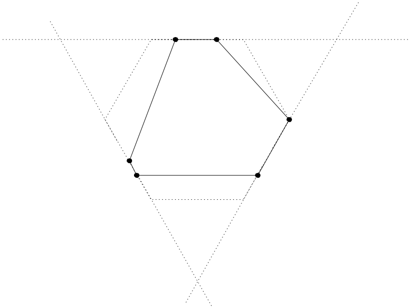

Property 2

A regular n-gon can be associated to an Ideal Polygon.

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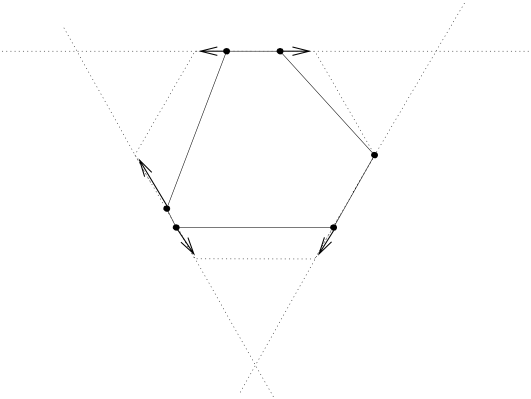

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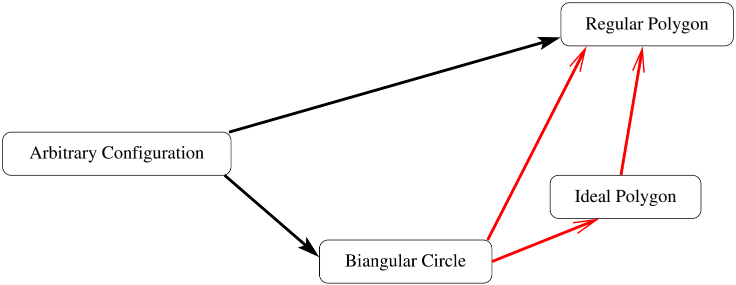

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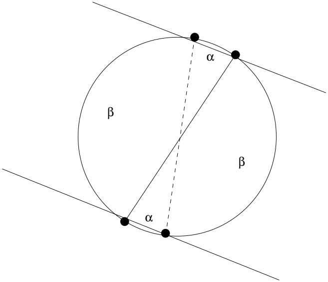
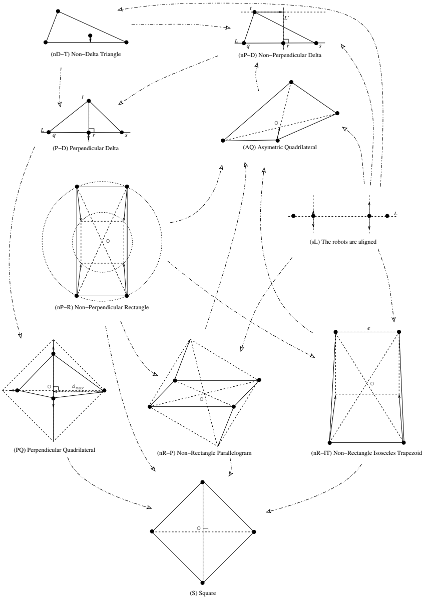


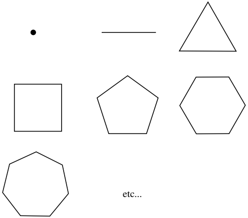
It works if $n \geq 5$ only

1. [Katreniak 2005] works if $n \geq 5$ only
2. Difficulty to find an invariant


(nD−T) Non−Delta Triangle
(nP−D) Non−Perpendicular Delta
(P−D) Perpendicular Delta
(AQ) Asymetric Quadrilateral
(sL) The robots are aligned
(nP−R) Non−Perpendicular Rectangle
(PQ) Perpendicular Quadrilateral
(nR−P) Non−Rectangle Parallelogram
(nR−IT) Non−Rectangle Isosceles Trapezoid
(S) Square

$n \leq 4$

Arbitrary Pattern Formation



Theorem

$\forall n$, the class of patterns that can be solved deterministically assuming robots devoid of sense of direction (Θ) is equal to the set of regular polygons (n-gons).