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Theoretical Computer Science 311 (2004) 165 – 197

Theoretical Computer Science

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On liveness and boundedness of asymmetric choice nets☆

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Received 17 September 2002; received in revised form 31 March 2003; accepted 16 June 2003
Communicated by G. Rozenberg

Abstract

This paper concerns two important techniques, characterization and property-preserving transformation, for verifying some basic properties of asymmetric choice Petri nets (AC nets). In the literature, a majority of the characterizations are for ordinary free choice nets. This paper presents many extended (from free choice nets) and new characterizations for four properties: liveness with respect to an initial marking, liveness monotonicity with respect to an initial marking, well-formedness, liveness and boundedness with respect to an initial marking. The nets involved are extended to homogeneous free choice nets, ordinary AC nets and homogeneous AC nets. This paper also investigates the transformation of merging a set of places of an ordinary AC net and proposes the conditions for it to preserve the siphon-trap-property (ST-property), liveness, boundedness and reversibility. The results are then applied to the verification of resource-sharing systems. At present, the major approaches for solving this problem are based on state machines or marked graphs and are not based on property preservation. Our approach extends the scopes of the underlying nets to AC nets and the verification techniques. It is found that the ST-property plays a very important role in many of the results. Furthermore, mainly through examples, the importance of the assumptions in the proposed characterizations and transformation and the limitation on further extensions are pointed out.

Keywords: Analysis; Asymmetric choice nets; Characterization; Liveness and boundedness; Well-formedness; Theory of Petri nets

☆ This research was financially supported by the Research Grant Council of Hong Kong under Grant CityU 1133/99E and CityU 1072/00E.
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0304-3975/$ - see front matter
doi:10.1016/S0304-3975(03)00359-1

1. Introduction

1.1. Verification through characterization and property-preserving transformation

Verification is the process of showing whether or not a design specification possesses the desirable properties and is free of the undesirable ones. Usually, whether a property is desirable or not depends on the objectives and requirements of the problem. For most problems, in the terminology of Petri nets, desirable properties include liveness, boundedness and reversibility, whereas undesirable properties include deadlock, storage space overflow, improper termination, etc.

For system designs specified in Petri nets, the major approaches for verification include reachability analysis, direct proving on the basis of definitions, mathematical programming, characterization and property-preserving transformation. Except for simple systems, the first three of these approaches are often either computationally intractable or too difficult. Characterization and property-preserving transformation are two extensively-used auxiliary techniques for verification. Briefly, a characterization of a property is a structural, logical or algebraic relationship associating the property with some others. If it is difficult to verify this property directly, it may be proved indirectly by verifying the other properties involved in the characterization [6,9,19,27,28]. A transformation preserving a property changes a net to another net so that the property under concern should not be created or destroyed. Hopefully, it is easier to carry out the verification process on the new net.

This paper is about characterization and property-preserving transformation concerning three basic properties (individually or in a combination), namely, liveness, boundedness and reversibility, essentially for asymmetric choice Petri nets (AC nets). Briefly, a system is live if all its operations are eventually executable, starting not only from its initial state but also from any reachable state. In particular, liveness implies the absence of deadlocks. A system is bounded if it has a finite number of states and is reversible if it can return to its initial state from any state. In the terminology of Petri nets, liveness requires the firability of every transition starting from any reachable marking, boundedness implies that the number of tokens existing in every place will not exceed a certain limit and reversibility means that the initial marking can be reached from any reachable marking. In general, these properties are independent and depend not only on the global structure of the Petri net but also on its initial marking.

There exist many characterizations for liveness and boundedness in the literature. They may be for liveness [3,4,7,9,16] or boundedness individually, or for a combination of them as a joint property [9,18,21,24–26]. This joint property is often investigated as two different problems: One concerns just the existence but not the actual value of an initial marking such that both properties hold for the net. If so, the net is said to be well-formed. Another is to determine whether both properties hold with respect to a specific initial marking.

At the present stage of the art, most of the characterizations of liveness and boundedness are for a scaled-down version of liveness or for special classes of Petri nets. For example, a general Petri net is structurally deadlock-free if it satisfies the ST-property (i.e., every siphon contains at least one trap) [7]. A marked graph is live iff every circuit is initially marked [9]. Two well-known characterizations, namely, Commoner's Theorem and the Rank Theorem, exist for free choice Petri nets. Commoner's Theorem states that a free choice net is live iff every siphon contains a marked trap [9]; whereas the Rank Theorem characterizes well-formedness of free choice nets in terms of some structural properties and a relation between the rank and the number of clusters of the Petri net [8,9].

Structurally, there may be more systems satisfying Commoner's property than the Rank-Theorem. However, the checking the former requires a higher order of complexity whereas checking the latter requires only polynomial time. This paper adopts the Commoner's approach (in fact, the ST-property). However, our study focuses on characterizations rather than complexity.

Through characterizations, well-formed free choice nets were shown satisfying many useful properties [9], such as coverability by minimal siphons, S-components and T-components, every minimal siphon being a trap itself, etc. Free choice nets also have many applications, such as workflow management systems [1]. With such promising features and applications for free choice nets, recent researchers have been trying to extend the existing characterizations to more general types of Petri nets. Naturally, AC nets are the next target. At present, the following characterizations for AC nets have been reported: (1) An ordinary AC (OAC) net is live if every siphon contains at least one trap marked by the initial marking [20]. (2) A homogeneous AC (HAC) net is live iff, for every reachable marking, every siphon contains a place whose marked value is not less the minimum weight of the outgoing arcs from the place [5]. This characterization is for the entire HAC net and not for individual transitions. This result has been extended to the liveness of a subset of transitions for OAC nets [16] and to individual transitions for HAC nets [17]. For more details of the above preview, see Tables 1–3 in Section 2.

In the literature, there have also been a lot of studies in transformations that preserve liveness and/or boundedness. For example, this has been studied for marked graphs [21] and free choice nets under reduction and synthesis [9,11–13]. Conditions for the preservation of 19 properties (including liveness and boundedness) under many forms of composition (such as sequential, choice, parallel, disable, recursive, etc.) were provided by Mak [19].

1.2. Preliminaries of Petri nets

This section outlines the definitions, terminology and properties as required in the paper.

A *weighted net* (or simply, *net*) is denoted by $N=(P,T,F,W)$, where P is a non-empty finite set of places, T is a non-empty finite set of transitions with $P\cap T=\emptyset$, $F\subseteq(P\times T)\cup(T\times P)$ is a flow relation and W is a weight function defined on the arcs, i.e., $W:F\to\{1,2,3,\ldots\}$. $N_1=(P_1,T_1,F_1,W_1)$ is called a *subnet* of N if $P_1\subseteq P$, $T_1\subseteq T$, $F_1=F\cap((P_1\times T_1)\cup(T_1\times P_1))$ and $W_1=W\,|\,F_1$, i.e., the restriction of W on F_1. The *pre-set* of x is defined as ${}^\bullet x=\{y\in P\cup T\mid(y,x)\in F\}$ and the *post-set* of x is defined as $x^\bullet=\{y\in P\cup T\mid(x,y)\in F\}$. Similarly, for any subset of $Y\subseteq P\cup T$, ${}^\bullet Y$ (resp., $Y^\bullet$) denotes the union of ${}^\bullet y$ (resp., $y^\bullet$) for all $y\in Y$. A net N is said to be *ordinary* and is denoted as $N=(P,T,F)$ if the weight of every arc is 1. The weight W is said to be *homogeneous* if, $\forall p\in P$, $\forall t_1,t_2\in p^\bullet$, $W(p,t_1)=W(p,t_2)$. A net $N=(P,T,F,W)$ is said to be *pure* or *self-loop-free* iff ${}^\bullet x\cap x^\bullet=\emptyset\ \forall x\in P\cup T$. In this article, we assume that all nets are pure.

The *incidence matrix* A of a pure net N is a $|P|\times|T|$ matrix whose element a_{ij} at row p_i and column t_j is denoted as follows:

$$a_{ij}=\begin{cases} w(t_j,p_i) & \text{if} \quad p_i\in t_j^\bullet,\\ -w(p_i,t_j) & \text{if} \quad p_i\in {}^\bullet t_j,\\ 0 & \text{otherwise.}\end{cases}$$

A *marking* of a net $N=(P,T,F,W)$ is a mapping $M:P\to\{0,1,2,\dots\}$. A place p is said to be *marked* by M if $M(p)>0$. For $P'\subseteq P$, P' is marked by M if $\exists p\in P'$ such that $M(p)>0$. $M(P')$ denotes the sum of $M(p)$ for all p in P'. A transition t is *enabled* or *firable* at a marking M if for every $p\in{}^\bullet t$, $M(p)\geqslant W(p,t)$. A transition t may be *fired* if it is enabled. Firing transition t results in changing the marking M to a new marking M', where M' *is* obtained by removing $W(p,t)$ tokens from each $p\in{}^\bullet t$ and by putting $W(t,p)$ tokens to every $p\in t^\bullet$. The process is denoted by $M[N,t\rangle M'$. If $M[N,t_1\rangle M_1[N,t_2\rangle\dots M_{n-1}[N,t_n\rangle M_n$, then $\sigma=t_1\dots t_n$ is called a *firing sequence* leading from M to M_n and is denoted as $M[N,\sigma\rangle M_n$. $R(N,M_0)$ denotes the set of all markings reachable from the *initial marking* M_0.

A transition t is said to be *live* in (N,M_0) iff, for any $M\in R(N,M_0)$, there exists $M'\in R(N,M)$ such that t can be fired at M'. (N,M_0) is said to be *live* iff every transition of N is live. A net N is said to be *structurally live* iff there exists a marking M_0 such that (N,M_0) is live. (N,M_0) is said to satisfy the *liveness monotonicity property* if (N,M) is live for any $M\geqslant M_0$. A place p is said to be *bounded* in (N,M_0) iff there exists a constant k such that $M(p)\leqslant k$ for all $M\in R(N,M_0)$. (N,M_0) is *bounded* iff every place of N is bounded. N is *structurally bounded* iff, for any marking M_0, (N,M_0) is bounded. N is said to be *well-formed* if there exists a marking M_0 such that (N,M_0) is live and bounded. (N,M_0) is said to be *reversible* iff $M_0\in R(N,M)$ $\forall M\in R(N,M_0)$. For $x\in P\cup T$, the *cluster* of x, denoted as $[x]$, is the smallest subset of $P\cup T$ satisfying three conditions: (1) $x\in[x]$; (2) if $p\in P\cap[x]$ then $p^\bullet\subseteq[x]$; and (3) if $t\in T\cap[x]$ then ${}^\bullet t\subseteq[x]$. N is said to satisfy the *rank-and-cluster property* iff the rank of its incidence matrix is less than the number of its clusters by 1.

A net $N=(P,T,F,W)$ is said to be *strongly connected* iff there exists a directed path from every node x to every node y. A net N is said to be *conservative* (resp., *consistent*) iff there exists a $|P|$-vector $\alpha>0$ such that $\alpha A=0$ (resp., $|T|$-vector $\beta>0$ such that $A\beta=0$), where A is the incidence matrix of N.

A *state machine* is a net $N=(P,T,F)$ such that $\forall t\in T:|{}^\bullet t|=|t^\bullet|=1$. A *marked graph* is a net $N=(P,T,F)$ such that $\forall p\in P:|{}^\bullet p|=|p^\bullet|=1$. A subnet $N_1=(P_1,T_1,F_1)$ of N is called an *S-component* of N iff N_1 is a strongly connected state machine and ${}^\bullet p\cup p^\bullet\subseteq T_1$ for every $p\in P_1$. N_1 is called a *T-component* of $N=(P,T,F)$ iff N_1 is a strongly connected marked graph and ${}^\bullet t\cup t^\bullet\subseteq P_1$ for every $t\in T_1$. N is said to be *free choice* (*FC*) iff $\forall p_1,p_2\in P: p_1^\bullet\cap p_2^\bullet\neq\emptyset\Rightarrow p_1^\bullet=p_2^\bullet$. N is said to be *asymmetric choice* (*AC*) iff $\forall p_1,\ p_2\in P: p_1^\bullet\cap p_2^\bullet\neq\emptyset\Rightarrow p_1^\bullet\subseteq p_2^\bullet$ or $p_2^\bullet\subseteq p_1^\bullet$. To avoid confusion, we use

OFC (resp., OAC) nets to denote ordinary FC (resp., AC) nets and HFC (resp., HAC) nets to denote homogeneous FC (resp., homogeneous AC) nets.

A non-empty set of places D is said to be a *siphon* (resp., *trap*) iff ${}^\bullet D \subseteq D^\bullet$ (resp., $D^\bullet \subseteq {}^\bullet D$). A siphon (resp., trap) is said to be *minimal* if it does not properly contain any other siphon (resp., trap). A siphon (resp., trap) is said to be *maximal* if it is not contained in any other siphons (resp., trap) except P.N is said to satisfy the *ST-property* if every siphon of N contains at least one trap.

For the convenience in referencing later, we quote from the literature [9,20] the following characterizations:

Property 1.1. An OFC net (N, M_0) is live iff every siphon contains a trap marked by M_0.

Property 1.2. An OAC net (N, M_0) is live if every siphon contains a trap marked by M_0.

Property 1.3. Let $N_1 = (P_1, T_1, F_1)$ be any S-component of an ordinary net (N, M_0). Then, $M(P_1) = M_0(P_1)$ for any $M \in R(N, M_0)$.

Property 1.4. A net N is structurally bounded iff there exists a $|P|$-vector $\alpha \geqslant 1$ such that $\alpha A \leqslant 0$, where A is the incidence matrix of N.

Property 1.5. If a Petri net (N, M_0) is live and reversible, then every trap of N is marked by M_0.

2. Summary of problems and results and organization of the paper

This paper first presents many new and extended characterizations for AC nets concerning three groups of properties. Group 1 characterizes liveness and liveness monotonicity as two separate properties whereas Groups 2 and 3 characterize them as a joint property. The paper then applies these results to the verification of resource-sharing systems via a place-merging transformation. Conditions on this transformation for preserving the ST-property, liveness, boundedness and reversibility are proposed. Furthermore, mainly through examples, analysis of these characterizations is provided and limitation of their extensions to other classes of Petri nets is pointed out.

More details of the three groups of characterizations and the application to resource-sharing are given below. For each group, a brief review of the major results existing in the literature and a preview of the results obtained in this paper are presented. For the application, the central idea of a property-preserving approach to verifying resource-sharing systems is pointed out. Note that the groups are not in a one-to-one correspondence relationship with the sections because some results belonging to one group can only be obtained after some results of another group have been obtained. Note also that, in the tables, Ax is either an extension to Fx or is a new characterization.

Table 1
Characterizations of liveness and liveness monotonicity as independent properties for FC nets and AC nets

Existing characterizations for liveness and liveness monotonicity		*Extended characterizations for liveness and liveness monotonicity*		*Proved or illustrated in*
F1a:	A live OFC net (N, M_0) satisfies the liveness monotonicity property [9].	A1a:	A live and bounded ST-OAC net (N, M_0) satisfies the liveness monotonicity property.	Corollary 5.1
F1b:	A live and bounded HFC net (N, M_0) satisfies the liveness monotonicity property [29].	A1b:	A live HFC net (N, M_0) satisfies the liveness monotonicity property.	Corollary 3.3
F2:	If a transition t of a marked S^3PR net (N, M_0) is not live, then there exist a siphon D and a marking $M \in R(N, M_0)$ such that $M(p) = 0 \forall p \in D$ [14]. (See Note 1 below Table 1.)	A2:	A transition t of a HAC net (N, M_0) is not live iff there exist a siphon D and a marking $M \in R(N, M_0)$ such that (1) $t \in D^\bullet$; and (2) $M(p) < W(p, t') \forall p \in D \forall t' \in p^\bullet$.	Theorem 3.1 (Corollary 3.1)
New characterizations for liveness and liveness monotonicity of HAC nets:				
A3:	A HAC net (N, M_0) satisfies the liveness monotonicity property iff, for every minimal siphon D of N, the D-induced subnet (N_D, M_D) is live. (Theorem 3.2)			
A4:	A HAC net (N, M_0) is live if, for every maximal siphon D of N, the D-induced subnet (N_D, M_D) is live. (Theorem 3.3)			

Note: S^3PR means 'system of simple sequential processes with resources'. Such a net can be created by integrating some simple sequential processes and merging their resources [14]. As A2, F1 is a characterization for a single transition. As Corollary 3.1, an extension to a F2 for the entire net of the class of nets S^3PGR2 is given in [23]. Details are omitted here.

Group 1. Characterizations of liveness or liveness monotonicity for AC nets with respect to a specific initial marking (Sections 3 and 5; Table 1).

This group contains characterizations of liveness or liveness monotonicity with respect to an initial marking for an AC net as two separate properties. Our new results include:

- three characterizations for liveness monotonicity, i.e., an extension from live OFC nets to live and bounded ST-OAC nets (A1a), an extension from bounded HFC nets to both bounded and unbounded HFC nets (A1b), and a new necessary and sufficient condition based on the liveness of the subnets induced by all the minimal siphons (A3),
- a new characterization (A2) for the non-liveness of individual transitions of a HAC net, and
- a new sufficient condition for the liveness of a HAC net (A4) based on the liveness of the subnets induced by all the maximal siphons.

Group 2. Characterizations of well-formedness for AC nets (Section 4; Table 2).

This group is to determine the conditions under which an initial marking will exist with respect to which an AC net is both live and bounded (as a joint property). Section 4 proposes many such characterizations extended from FC nets and discusses the limitation on further extension.

Table 2
Characterizations of well-formedness for FC nets and AC nets

Existing characterizations for well-formed FC nets	*Corresponding extensions for well-formed AC nets*	*Proved or illustrated in*
F5: The set of places of a well-formed OFC net can be covered by its minimal siphons [9].	A5: The set of places of a well-formed HAC net can be covered by its minimal siphons.	Theorem 4.1 Example 4.1
F6: A well-formed OFC net can be covered by its S-components [9].	A6: A well-formed ST-OAC net can be covered by its S-components.	Corollary 4.1 Example 4.2
F7: A well-formed OFC net can be covered by its T-components [9].	A7: A well-formed ST-OAC net may not be covered by its T-components.	Example 4.2
F8: A minimal siphon D of a well-formed OFC net is itself a trap and the D-induced subnet $(D, D^\bullet, F_D)$ is an S-component, where $F_D = F \cap ((D \times D^\bullet) \cup (D^\bullet \times D))$ [9].	A8: A minimal siphon D of a well-formed OAC net either does not contain any trap or is the only trap within itself. For the latter case, $(D, D^\bullet, F_D)$ is an S-component.	Theorem 4.2 Example 4.3
F9a: (Rank Theorem) An OFC net is well-formed iff it is connected, conservative, consistent and satisfies the rank-and-cluster property (RC-property) [9].	A9a: The sufficiency part of F9a had been extended to general Petri nets [9]. However, the necessity part is not valid. (A well-formed OAC net may be structurally unbounded and a well-formed ST-OAC net may not satisfy the RC-property.)	Example 4.4 Example 4.5
F9b: An HFC net is well-formed iff it is structurally live and structurally bounded [29].	A9b: An ST-OAC net is well-formed iff it is structurally bounded.	Theorem 4.3 Example 4.5

Some of our results involve the Rank Theorem. The Rank Theorem leads to two different necessary and sufficient conditions for checking well-formedness, one for OFC nets (F9a) and another for HFC nets (F9b). In the past, there have been a lot of efforts in extending such characterizations to more general nets than FC nets. While the sufficiency part of the Rank Theorem had been extended for general Petri nets [9], it was not sure whether its necessity part could also be extended or not. This section confirms that, indeed, it cannot be extended to general ordinary AC nets because a well-formed AC net may not be structurally bounded (Statement A9a and Example 4.5). On the other hand, we find that, by adding the ST-property as a constraint, a well-formed AC net, though still not necessarily satisfying the rank-and-cluster condition, is indeed structurally bounded. This leads to Characterization A9b.

Group 3. Characterizations of liveness, boundedness and reversibility for AC nets with respect to a given initial marking (Section 5; Table 3).

While Section 4 studies the well-formedness property that requires just the existence of an initial marking without concerning its actual value, Section 5 investigates the

Table 3
Characterizations of liveness and boundedness with respect to a marking for FC nets and AC nets

Existing characterizations for Live and bounded FC nets	*Corresponding extensions for live and bounded AC nets*	*Proved or illustrated in*
F10: A well-formed OFC net (N, M_0) is live and bounded iff every minimal siphon is marked by M_0 [9].	A10: A well-formed ST-OAC net (N, M_0) is live and bounded iff every minimal siphon is marked by M_0.	Theorem 5.1
F11: A live and bounded OFC net (N, M_0) is reversible iff every trap of N is marked by M_0 [9].	A11: F11 cannot be extended to ST-OAC nets.	Example 5.4
New characterizations for liveness and boundedness of HAC nets:		
A12: A HAC net (N, M) is live and bounded for any $M \geqslant M_0$ iff (1) N can be covered by minimal siphons, and (2) the subnet induced from every minimal siphon is live and bounded with respect to M_0. (Theorem 5.2)		
A13: A HAC net (N, M_0) is live and bounded if (1) N is covered by maximal siphons, and (2) the subnet induced from every maximal siphon is live and bounded with respect to M_0. (Theorem 5.3)		

conditions under which an AC net (N, M_0) is both live and bounded with respect to a special marking M_0, with some results concerning reversibility as a by-product. Two well-known characterizations (F10 and F11) of these properties exist for OFC nets. Section 5 shows that F10 (i.e., the characterization for liveness and boundedness) can be extended to ST-OAC nets (A10) whereas F11 (i.e., the characterization for reversibility) cannot (A11).

F10 and A10 both have their shortcoming: To apply them, the net N must be shown to be well-formed first. When applying the two new characterizations A12 or A13, it is not necessary to explicitly show whether N is well-formed or not. Instead, one just shows: (1) N is covered by optimal siphons; and (2) the subnet induced by every optimal siphon is live and bounded. In other words, the problem is reduced to solving the same problem for some subnets of N. This approach has two variations depending on whether optimal means "minimal" (Characterization A12) or "maximal" (Characterization A13). Usually, the number of maximal siphons is much smaller than the number of minimal siphons and it is easier to check for maximality than for minimality. On the other hand, the subnets induced from maximal siphons are naturally larger. However, note that Characterization A12 is a necessary and sufficient condition and implies liveness monotonicity for HAC nets, whereas Characterization A13 is only a sufficient condition and does not imply liveness monotonicity.

2.1. Application of results obtained in previous sections to the resource-sharing problem (Section 6)

Resource sharing is a very common and basic issue in system design. In manufacturing engineering, for example, robots and machining tools are shared among several machines or processes. There are many Petri-net-based approaches for verifying such systems. Most of them provide some conditions on the net for the system to be live, bounded and reversible. Some articles go further by adopting what is called a deadlock avoidance policy. For example, in [14,23], if the required conditions are not satisfied, extra places and transitions are added in such a way that the conditions will be fulfilled. In this paper, we use the property-preserving approach below: Each resource is represented as a place within a process that uses this resource. Whenever a resource becomes shared, the set of places involved will be merged. For verification purposes, this approach requires the merge process to preserve the properties under concern. The burden of this property-preservation approach lies on making sure that the merge process and the original net should both satisfy certain constraints.

Based on this approach, Agerwala et al. [3] proved the preservation of P-invariants under the 1-way merge. Narahari et al. [22] used invariants to study the absence of deadlocks, conservativeness and boundedness of the merged system. However, as far as we know, no *general* results concerning the preservation of liveness, the most important property of a system, have been reported in the literature.

In Section 6, the resource-sharing problem is formulated as a problem of merging several sets of places each into a single place. By applying the results obtained in the previous sections, we have obtained some simple conditions (to be imposed on the nets before merging) for ensuring that the merge will preserve asymmetric-free-choiceness, liveness, boundedness and reversibility. The advantage of our approach will be discussed in more detail in Section 6. Note that, since this paper just illustrates the application of our theoretical results as an example, a comprehensive review on the methods for solving the resource-sharing problem is not intended.

In Section 7, the characterizations obtained in this paper are classified according to the types of the nets and some concluding remarks are given.

3. Characterizing liveness and liveness monotonicity of HAC nets

This section begins with providing a new proof in Section 3.1 for an existing result concerning non-liveness of individual transitions of a HAC net with respect to a given initial marking. Then, based on this result, several characterizations for liveness and liveness monotonicity are derived in Section 3.2.

3.1. Checking non-liveness of individual transitions of HAC nets

A characterization concerning the non-liveness of an individual transition of a HAC net was reported in [17]. In this subsection, this characterization is restated as Theorem 3.1 but with a simpler proof. Theorem 3.1 and its Corollary 3.1 extend similar characterizations reported in [16] from OAC nets to HAC nets and the results reported in [14,23] for FMs and the class of nets S^3PGR2 (See Note 1 in Section 1 of this paper).

Lemma 3.1. *Let* (N, M_0) *be a HAC net. If* t *is the only non-live transition of* (N, M_0), *then there exist* $p \in {}^\bullet t$ *and* $M \in R(N, M_0)$ *such that* $\{p\}$ *is a siphon and* $M(p) < W(p, t)$.

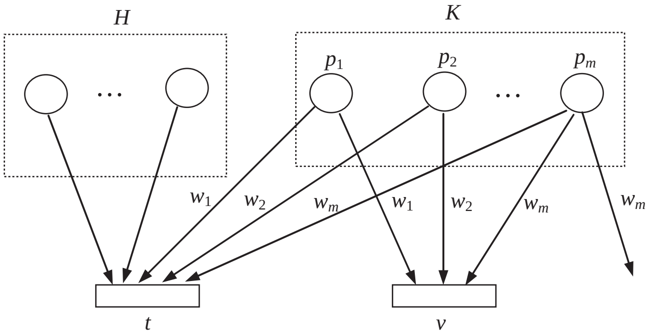

Fig. 1. Explanation for the proof of Lemma 3.1.

Proof. Obviously, ${}^\bullet t \neq \emptyset$ because, otherwise, t is live. In the following, it is shown that $\exists p \in {}^\bullet t$ such that ${}^\bullet p = \emptyset$. Then, ${}^\bullet p = \emptyset \subseteq p^\bullet$, i.e., $\{p\}$ is a siphon.

Let ${}^\bullet t$ be partitioned into two disjoint sets (Fig. 1): $H = \{p \mid p \in {}^\bullet t$ and $p^\bullet = \{t\}\}$ and $K = \{p | p \in {}^\bullet t$ and $p^\bullet \supset \{t\}\}$. Then, H and K cannot be both empty.

Since t is not live in (N, M_0), there exists $M_1 \in [N, M_0\rangle$ such that t is not enabled at any marking reachable from M_1. For each place $p \in H$ satisfying ${}^\bullet p \neq \emptyset$, since t cannot occur and all other transitions are live in (N, M_1), we can fire those transitions in ${}^\bullet p$ sufficiently often until p carries at least $W(p,t)$ tokens. Let M_2 be the marking reached. If $K = \emptyset$, then there exists at least one place p such that ${}^\bullet p = \emptyset$ because otherwise t will be enabled again. Assume that $K \neq \emptyset$, let $K = \{p_1, p_2, \ldots, p_m\}$, where $m \geqslant 1$. Since N is an AC net and $p_i^\bullet \cap p_j{}^\bullet \neq \emptyset$ for $i, j \in \{1, 2, \ldots, m\}$, without loss of generality, we may assume that $p_1^\bullet \subseteq p_2^\bullet \subseteq \cdots \subseteq p_m^\bullet$. Since $p_1^\bullet - \{t\} \neq \emptyset$, by the definition of K, $\exists v \in p_1^\bullet - \{t\}$. It follows from this assumption that $v \in p_i^\bullet$, for $i = 1, \ldots, m$. This implies that $K \subseteq {}^\bullet v$. Since v is live, starting with M_2, a marking M_3 which enables v will be reached. Otherwise, define $M_3 = M_2$.

Since M_3 does not enable t, there exists a place $p \in {}^\bullet t$ such that $M_3(p) < W(p,t)$. We have that $p^\bullet = \{t\}$ because otherwise $p \in K$ and thus $p \in {}^\bullet v$ and M_3 enables v. ${}^\bullet p = \emptyset$ because otherwise $M_3(p) \geqslant M_2(p) \geqslant W(p,t)$. □

Theorem 3.1. *Let* (N, M_0) *be a HAC net, where* $N = (P, T, F, W)$. *Then,* $t \in T$ *is non-live in* (N, M_0) *iff there exist a siphon* D *and a marking* $M \in R(N, M_0)$ *such that* (1) $t \in D^\bullet$; *and* (2) $\forall p \in D\ \forall t' \in p^\bullet : M(p) < W(p, t')$.

Proof. ($\Leftarrow$): Conditions (1) and (2) imply that there exist a siphon D and a marking $M \in R(N,\ M_0)$ at which, no $t' \in D^\bullet$, including t, can be enabled. Since ${}^\bullet D \subseteq D^\bullet$, no $t' \in {}^\bullet D$ can be enabled at M either. This implies that the number of tokens in every $p \in D$ remains unchanged and that t remains non-firable forever.
($\Rightarrow$): Since t is not live in (N, M_0), there exists a marking $M \in R(N, M_0)$ such that t is dead at M. We proceed by induction on the number of transitions that are not live at M.

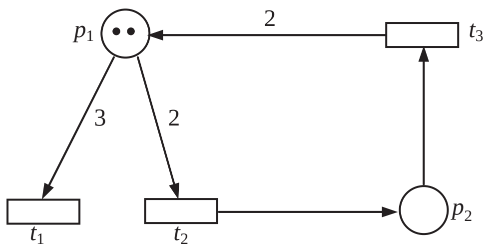

Fig. 2. An inhomogeneous AC net with non-live transition t_1.

If the number is 1, then the result follows from Lemma 3.1. Assume that the proposition holds for all nets with m non-live transitions. For a net (N, M_0) with $m+1$ non-live transitions, there is another transition u not live at (N, M). A marking M_1 from M is reached such that u is dead at M_1. This means that t and u are both not live in (N, M_1). Let N_t be the net after deleting t and all its associated arcs in N. It is obvious that N_t is still a HAC net. Since u cannot be enabled at any marking reachable from M_1, u is not live in (N_t, M_1). Since the number of transitions of N_t is m, there exist a siphon D_u in N_t satisfying ${}^\bullet u \cap D_u \neq \emptyset$ and $M_u \in R(N_t, M_1)$ such that $M_u(p) < W(p, t') \forall p \in D_u \forall t' \in p^\bullet$. Similarly, t is not live in (N_u, M_u), where N_u is the net after deleting u and all its associated arcs in N. Hence, there exist a siphon D_t in N_u satisfying ${}^\bullet t \cap D_t \neq \emptyset$ and $M_t \in R(N_u, M_u)$ such that $M_t(p) < W(p, t') \forall p \in D_t \forall t' \in p^\bullet$.

Since $M_t \in R(N_t, M_u)$, for every $p \in D_u : M_t(p) < W(p, t') \forall t' \in p^\bullet$. This implies that $M_t(p) < W(p, t') \forall p \in (D_t \cup D_u) \forall t' \in p^\bullet$. In N, since $t \in D_t^\bullet$ and $u \in D_u^\bullet$, ${}^\bullet(D_t \cup D_u) \subseteq {}^\bullet D_t \cup {}^\bullet D_u \cup \{u, t\} \subseteq D_t^\bullet \cup D_u^\bullet = (D_t \cup D_u)^\bullet$, i.e., $D_t \cup D_u$ is a siphon of N. It is obvious that $R(N, M_1) = R(N_t, M_1)$ and $R(N, M_u) = R(N_u, M_u)$. Hence, $M_t \in R(N, M_u) \subseteq R(N, M_1) \subseteq R(N, M) \subseteq R(N, M_0)$. □

Example 3.1. This example shows that, in Theorem 3.1, the homogeneity assumption is required for AC nets. Consider the inhomogeneous AC net N of Fig. 2 for which $R(N, M_0) = \{M_0, M_1\}$, where $M_0 = (2\ 0)$ and $M_1 = (0\ 1)$. $\{p_1, p_2\}$ is the only siphon of N and t_1 is the only non-live transition of (N, M_0). However, $M_0(p_1) = W(p_1, t_2) = 2$ and $M_1(p_2) = W(p_2, t_3) = 1$.

Characterization of non-liveness for individual transitions in Theorem 3.1 easily leads to an existing result (stated in Corollary 3.1 below) of similar characterization for an entire HAC net.

Corollary 3.1 (Barkaoui and Pradat-Peyre [5]). *A HAC net (N, M_0) is non-live iff there exist a siphon D and a marking $M \in R(N, M_0)$ such that $M(p) < W(p, t)$: $\forall p \in D \forall t \in p^\bullet$.*

3.2. Characterizing liveness monotonicity of HFC nets and HAC nets

To characterize liveness monotonicity of a HAC net, it is necessary to find all minimal siphons of the net. In general, it is time-consuming to determine whether a set of places is a minimal siphon or not. For AC nets, the simple characterization of siphons [17] as stated in Lemma 3.2 below, it is an extension from FC nets [9], may improve the efficiency.

Property 3.1 (Hack [15]). For a minimal siphon D of net N, the subnet $N' = (D, {}^\bullet D, {}^\bullet F)$ is strongly connected, where ${}^\bullet F = F \cap ((D \times {}^\bullet D) \cup ({}^\bullet D \times D))$.

Lemma 3.2. *For an AC net* $N = (P, T, F)$, *let* $D \subseteq P$ *and* $N' = (D, {}^\bullet D, {}^\bullet F)$. *Then*, D *is a minimal siphon iff* (1) N' *is strongly connected, and* (2) $|{}^\bullet t \cap D| = 1$ *for every* $t \in D^\bullet$.

Proof. ($\Leftarrow$): Since N' is strongly connected, for any $t \in {}^\bullet D$, there exists $p \in D$ such that $t \in p^\bullet \subseteq D^\bullet$. Hence, D is a siphon. Suppose that exists a siphon $D' \subset D$. Since N' is strongly connected, it is possible to find $p \in D - D'$, $p' \in D'$ and $t \in {}^\bullet D$ such that the arcs (p, t) and (t, p') belong to ${}^\bullet F$, i.e., $p \in {}^\bullet t$. Since $t \in {}^\bullet p' \subseteq {}^\bullet D' \subseteq (D')^\bullet$, there exists $q \in D'$ such that $q \in {}^\bullet t$. This contradicts with Condition (2) that p is the only input place of t in the entire D.
($\Rightarrow$): Suppose D is a minimal siphon. Condition (1) follows from Property 3.1. Suppose there exists $t \in D^\bullet$ such that ${}^\bullet t \cap D = \{p_1, p_2, \ldots, p_m\}$, where $m \geqslant 2$. Since N is an AC net, without loss of generality, we can assume that $p_1^\bullet \subseteq p_2^\bullet \subseteq \cdots \subseteq p_m^\bullet$. Since $p_m \in D - \{p_1\}$ and $p_1^\bullet \subseteq p_m^\bullet$, deleting p_1 from D does not reduce $D^\bullet$, i.e., $D^\bullet = (D - \{p_1\})^\bullet$. Hence, ${}^\bullet(D - \{p_1\}) \subseteq {}^\bullet D \subseteq D^\bullet = (D - \{p_1\})^\bullet$, i.e., $D - \{p_1\}$ is a siphon. This contradicts with the fact that D is minimal. $\square$

Definition 3.1 (induced subnet N_D). Let $N = (P, T, F, W)$ be a net. For $D \subseteq P$, $N_D = (D, D^\bullet, F_D, W_D)$ is called a *D-induced subnet* of N, where $F_D = F \cap ((D \times D^\bullet) \cup (D^\bullet \times D))$ and $W_D = W \mid F_D$.

Lemma 3.3 below follows from Lemma 3.2 and Definition 3.1.

Lemma 3.3. *Let* D *be a minimal siphon of an AC net* $N = (P, T, F, W)$. *Then, the D-induced subnet* N_D *is FC and its every transition has only one input place.*

The following lemma states a simple condition for liveness monotonicity for general nets.

Lemma 3.4. *For a net* $N = (P, T, F, W)$ *satisfying* $|{}^\bullet t| \leqslant 1 \, \forall t \in T$, *if* (N, M_0) *is live, then* (N, M) *is live for any* $M \geqslant M_0$.

Proof. Suppose there exists a marking $M \geqslant M_0$ such that (N, M) is not live. Since $|{}^\bullet t| \leqslant 1 \, \forall t \in T$, N is HFC. By Corollary 3.1, there exist a firing sequence σ, a siphon D and a marking M' such that $M[N, \sigma\rangle M'$ and $M'(p) < W(p, t) \forall p \in D \forall t \in p^\bullet$. In the following, we show by induction on $|\sigma|$ that there exist σ' and M_0' such that $M_0[N, \sigma'\rangle M_0'$ and $M_0'(p) < W(p, t) \forall p \in D \forall t \in p^\bullet$. This means that no $t \in D^\bullet$ be enabled again, contradicting with the fact that (N, M_0) is live.

Let $\sigma = t_1 t_2 \dots t_n$. If $|\sigma| = 0$, then $M' = M$. Let σ' be a null sequence. Then, $M_0' = M_0$ and $M_0[N, \sigma'\rangle M_0'$. Also, $M_0' = M_0 \leqslant M$ and $M_0'(p) = M'(p) < W(p,t) \forall p \in D \forall t \in p^\bullet$. Next, assume that, whenever $|\sigma| \leqslant m$, there exist σ', D' and M_0' such that $M_0[N, \sigma'\rangle M_0'$ and $M_0'(p) < W(p,t) \forall p \in D' \forall t \in p^\bullet$. For $|\sigma| = m+1$, since every transition has at most 1-input place and (N, M_0) is live, σ must contain a transition that is enabled at M_0. Suppose that $t_1, \dots, t_k$ are not enabled at M_0 but t_{k+1} is enabled at M_0. Then, there exists M_0'' such that $M_0[N, t_{k+1}\rangle M_0''.t_{k+1}$ is also enabled at M because $M \geqslant M_0$. Since those transitions of $({}^\bullet t)^\bullet$ are also enabled at M if t is enabled at M, this means that $({}^\bullet t_1 \cup {}^\bullet t_2 \cup \cdots \cup {}^\bullet t_k) \cap {}^\bullet t_{k+1} = \emptyset$. Hence, there exist M'' and M_0'' such that $M[N, t_{k+1}\rangle M''[N, t_1 \dots t_k t_{k+2} \dots t_{m+1}\rangle M'$. It is obvious that (N, M'') is not live. Since $M \geqslant M_0$, $M'' \geqslant M_0''$. By induction, there exist σ', D' and M_0' such that $M_0''[N, \sigma'\rangle M_0'$ and $M_0'(p) < W(p,t) \forall p \in D' \forall t \in p^\bullet$. That is, there exist $t_{k+1}\sigma'$, D' and M_0' such that $M_0[N, t_{k+1}\sigma'\rangle M_0'$ and $M_0'(p) < W(p,t) \forall p \in D' \forall t \in p^\bullet$. □

By the above results, we can derive another characterization for the liveness monotonicity of a HAC net in terms of the liveness of those subnets generated by its minimal siphons.

Theorem 3.2. *Let (N, M_0) be a HAC net. Then, (N, M) is live for any $M \geqslant M_0$ iff, for every minimal siphon D of N, the D-induced subnet (N_D, M_D) is live.*

Proof. ($\Leftarrow$): Suppose there exists a transition t which is not live in (N, M_0). By Theorem 3.1, there exist a siphon D', a transition sequence $\sigma \in T^*$ and $M' \in R(N, M_0)$ such that $t \in (D')^\bullet$, $M_0[N, \sigma\rangle M'$ and $M'(p) < W(p, t') \forall p \in D' \forall t' \in p^\bullet$. Let D be a minimal siphon contained in D' and σ_D be the restriction of σ over $D^\bullet$. Since ${}^\bullet D \subseteq D^\bullet$, the firing of any transition in $T \backslash D^\bullet$ does not influence the token distribution in D. Hence, σ_D can also be fired in (N_D, M_D) with the result that $M_D[N_D, \sigma_D\rangle M_D'$ and $M_D'(p) < W(p, t') \forall p \in D \forall t' \in p^\bullet$, where M_D is the restriction of M_0 over D. By Theorem 3.1, t is not live in (N_D, M_D), contradicting with the fact that (N_D, M_D) is live.

For any minimal siphon D of N and any $M \geqslant M_0$, let $M_D'' = M \mid D$. Hence, $M_D'' \geqslant M_D$. By Lemma 3.3, N_D satisfies $|{}^\bullet t| \leqslant 1 \forall t \in T_D$. By Lemma 3.4, (N_D, M_D'') is live. Hence, (N, M) is live.

($\Rightarrow$): Suppose that there exists a minimal siphon D such that (N_D, M_D) is not live. Since (N_D, M_D) is not live and D is minimal siphon, according to Theorem 3.1, there exist a firing sequence σ and a reachable marking M_D' such that $M_D[N_D, \sigma\rangle M_D'$ and $M_D'(p) < W(p,t) \forall p \in D \forall t \in p^\bullet$. Adding enough tokens in $P - D$, if necessary, a new marking $M \geqslant M_0$ is obtained such that $M[N, \sigma\rangle M'$, where $M_D = M \mid D$ and $M_D' = M' \mid D$. Hence, $M'(p) < W(p,t) \forall p \in D \forall t \in p^\bullet$. By Theorem 3.1, (N, M) is not live—a contradiction. □

There is a recent result [30] concerning the liveness monotonicity of OAC nets. Since it is not yet very well known and its proof is quite complex, it is restated in Corollary 3.2 with a new proof.

Corollary 3.2 (Zhen and Lu [30]). *An OAC net (N, M) is live for any $M \geqslant M_0$ iff M_0 marks a trap of every minimal siphon.*

Proof (new). (⇒): By Theorem 3.2, for every minimal siphon D, the D-induced subnet (N_D, M_D) is live. According to Lemma 3.3, N_D is FC. Hence, it follows from Property 1.1 that D contains a trap marked by M_0. (⇐): Suppose M_0 marks a trap of every minimal siphon. By Property 1.1, the induced subnet of every minimal siphon is live. It follows from Theorem 3.2 that (N, M) is live for any $M \geqslant M_0$. □

When using Theorem 3.2 to determine the liveness of a HAC net, we have to check the liveness of the *D-induced subnet* for every minimal siphon D. In practice, the number of minimal siphons may be huge. In the literature, minimal siphons have been applied in many occasions [8,9]. In the following, we propose a similar but new characterization based on maximal siphons, i.e., siphons not properly contained in another siphon except the entire net. Applying the latter has certain advantages. First, using maximal siphons always succeeds whenever using minimal siphons succeeds but using maximal siphons may succeed even using minimal siphons fails (see Example 5.1). Secondly, in general, the number of maximal siphons would be smaller than the number of minimal siphons. Thirdly, it is easier to determine the maximality than minimality of a siphon. However, we can only use minimal siphons to check liveness monotonicity.

Theorem 3.3. *A HAC net (N, M_0) is live if the D-induced subnet (N_D, M_D) is live for every maximal siphon D of N.*

Proof. Suppose there exists a transition t which is not live in (N, M_0). By Theorem 3.1, there exist a siphon D' satisfying $t \in (D')^\bullet$, a transition sequence $\sigma \in T^*$ and $M' \in R(N, M_0)$ such that $M_0[N, \sigma\rangle M'$ and $M'(p) < W(p, t')\, \forall p \in D'\, \forall t' \in p^\bullet$. Since there exists a maximal siphon D which contains D', let σ_D be the restriction of σ over $D^\bullet$. Since ${}^\bullet D \subseteq D^\bullet$, the firing of any transition in $T \backslash D^\bullet$ does not influence the token distribution in D. Hence, σ_D can also be fired in (N_D, M_D), where M_D is the restriction of M_0 over D, with the result that $M_D[N_D, \sigma_D\rangle M'_D$ and $M'_D(p) < W(p, t')\, \forall p \in D'\, \forall t' \in p^\bullet$. By Theorem 3.1, t is not live in (N_D, M_D), contradicting with the fact that (N_D, M_D) is live. □

Teruel and Silva [29] proved that a bounded HFC net (N, M_0) satisfies the liveness monotonicity property. Corollary 3.3 below extends this result to include unbounded HFC nets.

Corollary 3.3. *If an HFC net (N, M_0) is live, then (N, M) is live for any $M \geqslant M_0$.*

Proof. We first prove that, for every minimal siphon D of N, the D-induced subnet (N_D, M_D) is live. Suppose that there exists a minimal siphon D such that the D-induced subnet (N_D, M_D) is not live. By Corollary 3.1, there exist a firing sequence σ, a minimal siphon D' and a marking M'_D such that $M_D[N_D, \sigma\rangle M'_D$ and $M'_D(p) < W(p, t)\, \forall p \in D'\, \forall t \in p^\bullet$. If $D' \subset D$, the D' is also a siphon of N, contradicting with the fact that D is minimal. Hence, $D' = D$. In the following, we show by induction on $|\sigma|$ that there exist σ' such that $M_0[N, \sigma'\rangle M'$ and $M'(p) < W(p, t)\, \forall p \in D\, \forall t \in p^\bullet$. This means that no $t \in D^{\bullet}$ can be enabled again in N, contradicting with the fact that (N, M_0) is live.

Let $\sigma = t_1 t_2 \ldots t_n$. If $|\sigma| = 0$, then $M' = M_0$ and $\sigma' = \emptyset$. Thus $M_0(p) = M_0'(p) < W(p,t)$ $\forall p \in D \forall t \in p^{\bullet}$. Assume that, whenever $|\sigma| \leqslant m$, there exist σ' and M' such that $M_0[N, \sigma'\rangle M'$ and $M'(p) < W(p,t) \forall p \in D \forall t \in p^{\bullet}$. For $|\sigma| = m+1$, consider the following two cases.

Case 1 (*There exists* $u \in D^{\bullet}$ *such that* u *is enabled at* M_0): Then, in $(N_D, M_D), u$ is enabled at M_D. In N, let $p \in D$ and $u \in p^{\bullet} = \{v_1, v_2, \ldots, v_l\}$. Since (N, M_0) is live, by Corollary 3.1, for every minimal siphon D' and every marking M reachable from M_0, there exists a $p \in D'$ such that $M(p) \geqslant W(p,t) \forall t \in p^{\bullet}$. But $M_D'(p) < W(p,t) \forall p \in D \forall t \in p^{\bullet}$, this means that $\{t_1, t_2, \ldots, t_{m+1}\} \cap \{v_1, v_2, \ldots, v_l\} \neq \emptyset$. Suppose $t_1, \ldots, t_k$ are not enabled at M_0 but t_{k+1} is enabled at M_0. According the definition of HFC nets, those transitions of $({}^{\bullet}t)^{\bullet}$ are also enabled at M if t is enabled at M. This means that $({}^{\bullet}t_1 \cup {}^{\bullet}t_2 \cup \cdots \cup {}^{\bullet}t_k) \cap {}^{\bullet}t_{k+1} = \emptyset$. Hence, there exist M'' such that $M_0[N, t_{k+1}\rangle M''$ and $M_D[N_D, t_{k+1}\rangle M_D''[N_D, t_1 t_2 \ldots t_k t_{k-1} \ldots t_m t_{m+1}\rangle M_D'$, where $M_D'' = M''|D$. By the induction hypothesis, there exist σ' and M' such that $M_0[N, t_{k+1}\rangle M''[N, \sigma'\rangle M'$ and $M'(p) < W(p,t)$ $\forall p \in D \forall t \in p^{\bullet}$.

Case 2 (*No transition* $u \in D^{\bullet}$ *such that* u *is enabled at* M_0): Since (N, M_0) is live, for some $u \in D^{\bullet}$, $\exists \sigma_1$ and M_1 such that σ_1 does not contain any transition in $D^{\bullet}$, $M_0[N, \sigma_1\rangle M_1$ and u is enabled at M_1. Since ${}^{\bullet}D \subseteq D^{\bullet}$, the firing of any transition in $T \backslash D^{\bullet}$ does not influence the token distribution in D. This implies that $M_D = M_1 \mid D$. This becomes Case 1.

By Theorem 3.2, (N, M) is live for any $M \geqslant M_0$. □

4. Characterizing well-formedness for AC nets

This section extends several existing characterizations of well-formedness from FC nets to AC nets (Table 2). However, before providing the formal proofs for them, these characterizations will be explained in a few examples. They show clearly the important role the ST-property plays in characterizing these properties for AC nets. They also point out the limitation for further extension. For convenience, for the rest of the paper, we call an OAC net satisfying the ST-property an ST-OAC net.

Example 4.1 (for illustrating Characterization A5 of Table 2, i.e., Theorem 4.1). The HAC net in Fig. 3 is well-formed. According to Theorem 4.1, its places can be covered by minimal siphons, that is, by $\{p_1, p_2\}$ and $\{p_3, p_4\}$. Note that the coverability property of Theorem 4.1 is not valid without the boundedness assumption. For example, for the live but unbounded OAC net in Fig. 4, p_5 is not covered by the unique minimal siphon $\{p_1, p_2, p_3, p_4\}$.

Example 4.2 (for illustrating Characterization A6 and Statement A7 of Table 2, i.e., Corollary 4.1). The well-formed AC net N in Fig. 5 has two minimal siphons $D_1 = \{p_1, p_2, p_4\}$ and $D_2 = \{p_3, p_5\}$. Both are traps themselves. N is covered by the

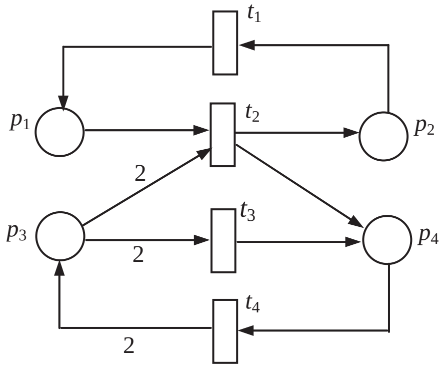

Fig. 3. A HAC net covered by its minimal siphons $\{p_1, p_2\}$ and $\{p_3, p_4\}$.

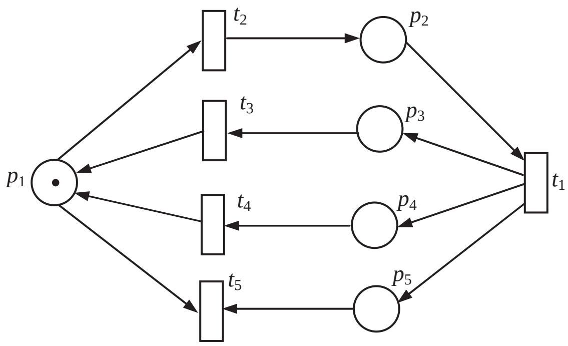

Fig. 4. A live but unbounded OAC net in which p_5 is not covered by any minimal siphon.

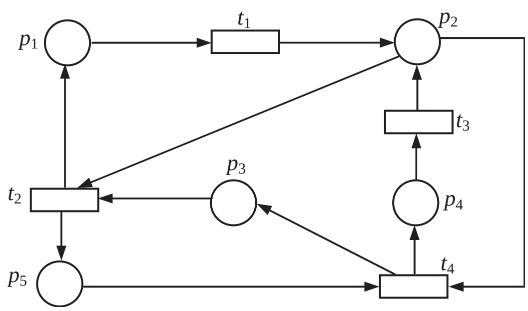

Fig. 5. A well-formed ST-OAC net covered by S-components but not by T-components.

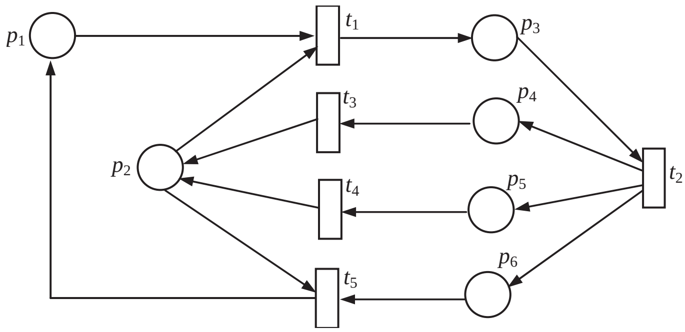

Fig. 6. A well-formed OAC net for illustrating Characterization A8.

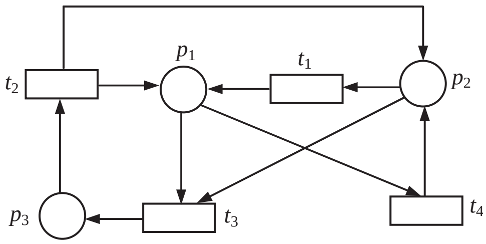

Fig. 7. A well-formed general Petri net showing the limitation on Characterization A8.

two S-components $(D_1,(D_1)^{\bullet},F_{D1})=(\{p_1,p_2,p_4\},\ \{t_1,t_3,t_4\},\ F_{D1})$ and $(D_2,(D_2)^{\bullet},F_{D2})=(\{p_3,p_5\},\{t_2,t_4\},\ F_{D2})$. On the other hand, N cannot be covered by T-components since it does not have any T-component. That is, Characterization F7 cannot be extended to OAC nets, even if they satisfy the ST-property (Statement A7).

Example 4.3 (for illustrating Characterization A8 of Table 2, i.e., Theorem 4.2). The OAC net N in Fig. 6 is well-formed since N is live and bounded for $M_0=(1\ 1\ 0\ 0\ 0\ 0)$. The minimal siphon $D_1=\{p_2,p_3,p_4,p_5\}$ does not contain any trap while the minimal siphon $D_2=\{p_1,p_3,p_6\}$ is also the only trap within itself. The D_2-induced subnet $(D_2,(D_2)^{\bullet},F_{D2})=(\{p_1,p_3,p_6\},\ \{t_1,t_2,t_5\},\ F_{D2})$ is an S-component. Characterization A8 is not valid for well-formed general Petri nets. For example, the net N in Fig. 7 is well-formed because N is live and bounded for $M_0=(1\ \ 1\ \ 0)$ but is not an OAC net because $p_1^{\bullet}=\{t_3,t_4\}$ and $p_2^{\bullet}=\{t_1,t_3\}$. Its unique minimal siphon $D=\{p_1,p_2,p_3\}=P$ is also a trap but its D-induced subnet (i.e., N) is not an S-component.

Example 4.4. (for illustrating Statement A9a of Table 2). A well-formed OAC net that does not satisfy the ST-property may not be structurally bounded. The OAC net N in Fig. 8 [27] is well-formed for the initial marking $(1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0)$ but does not satisfy the ST-property because the siphon $D=\{p_3,p_5,p_6,p_{10}\}$ does not contain any

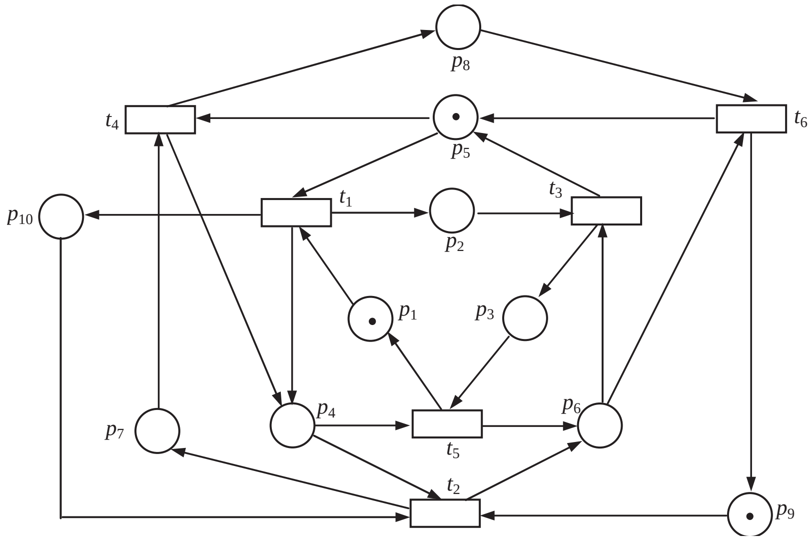

Fig. 8. A well-formed OAC net that is not structurally bounded [27].

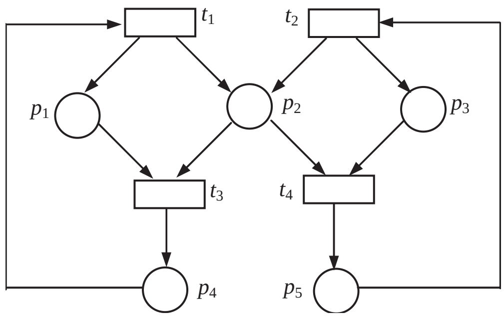

Fig. 9. A well-formed ST-OAC net that does not satisfy the RC-property.

trap. N is not structurally bounded because p_{10} becomes unbounded when firing $t_1t_3t_5$ infinitely many times from $(1\ 0\ 0\ 0\ 1\ 1\ 0\ 0\ 1\ 0)$.

Example 4.5. (for illustrating Characterization A9b of Table 2, i.e., Theorem 4.3). Consider the ST-OAC net N in Fig. 9, where all its minimal siphons $D_1=\{p_1, p_4\}$, $D_2=\{p_3, p_5\}$ and $D_3=\{p_2, p_4, p_5\}$ are also traps.

Since $\alpha V=0$ for $\alpha=(1\ \ 1\ \ 1\ \ 2\ \ 2)\geqslant 1$ and the following incidence matrix it follows from Property 1.4 that N is structurally bounded. By Theorem 4.3, N is well-formed. Next, N has 3 clusters $\{p_1, p_2, p_3, t_3, t_4\}$, $\{p_4, t_1\}$ and $\{p_5, t_2\}$ but the rank of V is 3.

Hence, N does not satisfy the rank-and-cluster property.

$$V = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 1 & 1 & -1 & -1 \\ 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

The theorems developed below support the characterizations listed in Table 2.

Theorem 4.1. *Let $N=(P,T,F,W)$ be a well-formed HAC net. Then, for every $p\in P$, there exists a minimal siphon of N that contains p.*

Proof. Suppose that there exists $p\in P$ that is not contained in any minimal siphon of N. Since N is well-formed, there exist M_0 such that (N,M_0) is live and bounded. Suppose, at $M_1\in R(N,M_0)$, p has the maximum valve $M_1(p)$ among all markings in $R(N,M_0)$. Let $M_1=M_2+M_p$, where $M_2(p)=0$ and $M_2(q)=M_1(q)$ for $q\neq p; M_p(p)=M_1(p)$ and $M_p(q)=0$ for $q\neq p$. Note that $M_1(q)\geqslant M_2(q)\forall q\in P$. We shall show that (N,M_2) is live. Suppose transition t is not live in (N,M_2). By Theorem 3.1, there exist a minimal siphon D of N, a marking M_2' and a firing sequence σ such that $M_2[N,\sigma\rangle M_2'$ and that $M_2'(q)<W(q,t')\forall q\in D\forall t'\in q^{\bullet}$. Since $M_1\geqslant M_2,\sigma$ can also be fired at M_1 and there exists M_1' such that $M_1[N,\sigma\rangle M_1'$, where $M_1'=M_2'+M_p$. Since $p\notin D, M_1'(q)=M_2'(q)<W(q,t')\forall q\in D\forall t'\in q^{\bullet}$, implying that (N,M_1) is not live. This contradicts with the fact that $M_1\in R(N,M_0)$ and (N,M_0) is live. Since (N,M_2) is live, there exist a firing sequence δ and M_3 such that $M_2[N,\delta\rangle M_3$ and $M_3(p)>0$. Obviously, $M_2[N,\delta\rangle M_3$ implies $(M_2+M_p)[N,\delta\rangle(M_3+M_p)$, i.e., $M_1[N,\delta\rangle(M_3+M_p)$. At p, the reached marking has a value $M_3(p)+M_1(p)>M_1(p)$. This contradicts with the fact that $M_1(p)$ is maximum. □

The minimal siphons of a well-formed OAC net satisfy several properties stated in the following Theorem 4.2.

Theorem 4.2. *Let D be a minimal siphon of a well-formed OAC net N. Then, D either does not contain any trap or is the only trap within itself such that the D-induced subnet $(D,D^{\bullet},F_D)$ is an S-component.*

Proof. Example 4.3 shows that D may not contain any trap. We shall show that, if D contains a trap S but $S\subset D$, then, S will be unbounded—a contradiction. Hence, $S=D$.

Since $S^{\bullet}\subseteq{}^{\bullet}S,{}^{\bullet}S$ includes all those transitions that will affect the token distribution within S and, thus, should be considered. Consider two cases: *Case* 1 ($t\in S^{\bullet}$): Since $t\in S^{\bullet}\subseteq{}^{\bullet}S\subseteq{}^{\bullet}D$, by Lemma 3.2, $|{}^{\bullet}t\cap S|\leqslant|{}^{\bullet}t\cap D|=1\forall t\in S^{\bullet}$. By the definition of a trap, $|t^{\bullet}\cap S|\geqslant 1\forall t\in S^{\bullet}$. Together, this means that firing any $t\in S^{\bullet}$ will take away at most one token from S but put at least one token back into S. That is, firing any t in $S^{\bullet}$ will not reduce the number of tokens in S. *Case* 2 ($t\in{}^{\bullet}S-S^{\bullet}$): Since t is an input transition but not an output transition of S, firing t will strictly increase the number of tokens in S. Since D is minimal, S cannot be a siphon, i.e., ${}^{\bullet}S \supset S^{\bullet}$. Hence, there exists at least one $t \in {}^{\bullet}S - S^{\bullet}$. Since (N, M_0) is live, Cases 1 and 2 together imply that firing a sequence that includes infinitely many t in $\in {}^{\bullet}S - S^{\bullet}$ will make S become unbounded.

By Lemma 3.2, $(D, {}^{\bullet}D, F_D)$ is strongly connected and $|{}^{\bullet}t \cap D| = 1\, \forall t \in {}^{\bullet}D$. Since D is also a trap, it follows from the definition of a trap that $|t^{\bullet} \cap D| \geqslant 1\, \forall t \in D^{\bullet} = {}^{\bullet}D$. If $\exists t \in D^{\bullet}$ such that $|t^{\bullet} \cap D| > 1$, then firing t each time will increase the tokens of N by a number equal to $|t^{\bullet} \cap D| - |{}^{\bullet}t \cap D| > 0$. Since (N, M_0) is live, t can be fired repeatedly, rendering D unbounded. Hence, $|{}^{\bullet}t \cap D| = |t^{\bullet} \cap D| = 1$, implying that $(D, D^{\bullet}, F_D)$ is an S-component. □

Corollary 4.1. *A well-formed ST-OAC net can be covered by S-components.*

Proof. By Theorem 4.1, a well-formed OAC net is covered by a finite set of minimal siphons. If the net also satisfies the ST-property, then, by Theorem 4.2, each of these minimal siphons is embedded within an S-component. □

While HFC nets had been shown to be structurally bounded [29], Example 4.4 shows that a well-formed OAC net may not be so. Theorem 4.3 below extends this property to ST-OAC nets.

Theorem 4.3. *An ST-OAC net N is well-formed iff it is structurally bounded.*

Proof ($\Leftarrow$). Let M_0 mark a trap of every siphon of net N. By Property 1.2, (N, M_0) is live. Since N is structurally bounded, (N, M_0) is bounded. Hence, N is well-formed.

($\Rightarrow$): By Corollary 4.1, N can be covered by a set of S-components $\{N_1, \ldots, N_m\}$, where $N_i = (P_i, T_i, F_i)$, $i = 1, \ldots, m$. Suppose M_0 is an arbitrary initial marking of N. By Property 1.3, $M(P_i) = M_0(P_i)\, \forall M \in R(N, M_0)$, $i = 1, \ldots, m$. This means $M(p) \leqslant M(P_1) + \cdots + M(P_m) = M_0(P_1) + \cdots + M_0(P_m)\, \forall p \in P$ and $\forall M \in R(N, M_0)$, i.e., (N, M_0) is bounded. Hence, N is structurally bounded. □

5. Characterizing liveness and boundedness of an AC net with respect to an initial marking

This section studies the conditions under which a given initial marking M_0 will render an AC net (N, M_0) both live and bounded (Table 3). The following two cases of N will be considered.

Case 1. Checking liveness and boundedness for (N, M_0), where N is a well-formed ST-OAC net.

In this case, N is known satisfying the ST-property and well-formedness in advance. An exact extension of a well-known characterization from OFC nets [9,18] to ST-OAC nets is presented in Theorem 5.1 below.

Theorem 5.1. *For a well-formed ST-OAC net N, (N, M_0) is live and bounded iff M_0 marks every minimal siphon of N.*

Proof (⇒). Suppose N has a minimal siphon D not marked by M_0. Then, D will not be marked by any $M \in R(N, M_0)$. This means that all transitions in $D^\bullet$ are not live in (N, M_0)—a contradiction.
(⇐): Since N is a well-formed ST-OAC net, by Theorem 4.2, every minimal siphon is a trap itself and is therefore marked by M_0. Hence, (N, M_0) is live by Property 1.2. By Theorem 4.3, N is structurally bounded. This implies that (N, M_0) is bounded. □

Case 2. Checking liveness and boundedness for a HAC net (N, M_0) without knowing whether N is well-formed or not.

In this case, we provide a new characterization with two variations, one based on minimal siphons (Theorem 5.2) and another on maximal siphons (Theorem 5.3).

Theorem 5.2. *A HAC net (N, M) is live and bounded for any $M \geqslant M_0$ iff the following two conditions hold*:
(1) *Every place p of N is covered by a minimal siphon.*
(2) *For every minimal siphon D, (N_D, M_D) is live and bounded, where N_D is the D-induced subnet and $M_D = M_0 \mid D$.*

Proof (⇐). First, we shall show that (N, M) is live and bounded for $M = M_0$.
By Condition (2) and Theorem 3.2, (N, M_0) is live. Suppose that (N, M_0) is not bounded at place p. By Condition (1), there exists a minimal siphon D containing p. Since the D-induced subnet (N_D, M_D) is bounded, there exists $M_1 \in R(N_D, M_D)$ such that $M_1(p)$ has the maximum valve. Since p is not bounded in (N, M_0), there exist a firing sequence σ and a reachable marking M' such that $M_0[N, \sigma\rangle M'$ and $M'(p) > M_1(p)$. It is obvious that $\sigma' = \sigma|D^\bullet$ can be fired in (N_D, M_D). Then, there exists a marking M'_D such that $M_D[N_D, \sigma'\rangle M'_D$ but $M'_D(p) = M'(p) > M_1(p)$, this contradicts with the fact that $M_1(p)$ is the maximum valve in (N_D, M_D).
Next, for any $M \geqslant M_0$ and any minimal siphon D of N, let $M'_D = M|D$. By Lemma 3.3, the induced subnet N_D is an HFC net. Since (N_D, M_D) is live and bounded and $M'_D \geqslant M_D$, it follows from Corollary 3.3 that (N_D, M'_D) is also live. By Characterization F9b, (N_D, M'_D) is bounded. By applying the above proof process on (N_D, M'_D), it follows that (N, M) is live and bounded.
(⇒): Since (N, M_0) is live and bounded, N is well-formed. By Theorem 4.1, every place p of N is covered by a minimal siphon, i.e., Condition (1) holds.
By Theorem 3.2, for any minimal siphon D, (N_D, M_D) is live. Suppose that there exists a minimal siphon D such that (N_D, M_D) is unbounded, then, there exists $p_0 \in D$ that is unbounded in (N_D, M_D). Hence, for an arbitrary positive integer k, there exist a firing sequence σ and a reachable marking M'_D such that $M_D[N_D, \sigma\rangle M'_D$ and $M'_D(p_0) > k$. Hence, there exists $M \geqslant M_0$ and M' such that $M[N, \sigma\rangle M'$ and $M'(p_0) > k$, contradicting that fact (N, M) is bounded for any $M \geqslant M_0$. Hence, for every minimal siphon D, (N_D, M_D) is live and bounded, where N_D is the D-induced subnet and $M_D = M_0 \mid D$, i.e., Condition (2) holds. □

It is well known that a live and bounded FC net satisfies the liveness monotonicity property [9,18]. The following corollary extends this result to ST-OAC nets.

Corollary 5.1. *If (N, M_0) is a live and bounded ST-OAC net, then (N, M) is live and bounded for any $M \geqslant M_0$.*

Proof. By Theorem 4.3, N is structurally bounded. For any minimal siphon D, since (N, M_0) is live, D is marked by M_0. Since N is well-formed and ST-OAC, by Theorem 4.2, D is a trap. This means that D is a trap marked by M_0. By Lemma 3.3, N_D is an OFC net, where N_D is the D-induced subnet. By Property 1.1, (N_D, M_D) is live, where $M_D = M_0|D$. For any $M \geqslant M_0$, since $M'_D \geqslant M_D$, by Characterization F1a, (N_D, M'_D) is live, where $M'_D = M|D$. It follows from that Theorem 3.2 that (N, M) is live. □

Theorem 5.3. *A HAC net (N, M_0) is live and bounded if the following two conditions hold*:

(1) *Every place p of N is covered by a maximal siphon.*
(2) *For every maximal siphon D, (N_D, M_D) is live and bounded, where N_D is the D-induced subnet and $M_D = M_0 \mid D$.*

Proof. By Theorem 3.3, (N, M_0) is live. Assume that there exists a place p that is not bounded. By Condition (1), there exists a maximal siphon D containing p. Since the D-induced subnet (N_D, M_D) is bounded, there exists $M_1 \in R(N_D, M_D)$ such that $M_1(p)$ has the maximum valve. Since p is not bounded in (N, M_0), there exist a firing sequence σ and a marking M' such that $M_0[N, \sigma\rangle M'$ and $M'(p) > M_1(p)$. It is obvious that $\sigma' = \sigma|D^\bullet$ can be fired in (N_D, M_D). Hence, there exists a marking M'_D such that $M_D[N_D, \sigma'\rangle M'_D$ but $M'_D(p) = M'(p) > M_1(p)$. This contradicts with the fact that $M_1(p)$ is the maximum valve in (N_D, M_D). □

5.1. Discussion on applying the above theorems to checking liveness and boundedness of AC nets

Theorem 5.1 requires first checking if the net is well-formed and satisfies the ST-property. If they are confirmed, checking the other conditions needs only polynomial time [18]. However, such checking is not needed when applying Theorems 5.2 and 5.3. In fact, these two theorems essentially reduce the problem for one bigger net to several similar problems for smaller subnets (i.e., the D-induced subnets). Commoner's Theorem can be applied to prove the liveness and boundedness of these D-induced nets because they are FC nets. Note: As shown in Table 4, Theorem 5.2 has more functions than Theorem 5.3. However, Theorem 5.3 has its advantages too. Note that the conditions of Theorem 5.2 imply the conditions of Theorem 5.3 but the converse is not true. This implies that, when checking liveness and boundedness, Theorem 5.3 may succeed even if Theorem 5.2 fails. This point is illustrated in Example 5.1 below.

Example 5.1. This example shows that, while Theorem 5.2 fails, Theorem 5.3 succeeds in confirming the liveness and boundedness of a HAC net. It also shows that Theorem 5.3 fails to make any conclusion about live monotonicity. The OAC net (N, M_0) in Fig. 10 is live and bounded. However, the subnet $(N_D, M_D) = ((\{p_1, p_3, p_5, p_6\}, \{t_1, t_2,$

Table 4
Comparison of Theorems 5.2 and 5.3

Functions	Theorem 5.2	Theorem 5.3
Applicable for checking liveness and boundedness	Yes	Yes
Applicable for checking the liveness monotonicity	Yes	No
Applicable as a sufficient condition	Yes	Yes
Applicable as a necessary condition	Yes	No

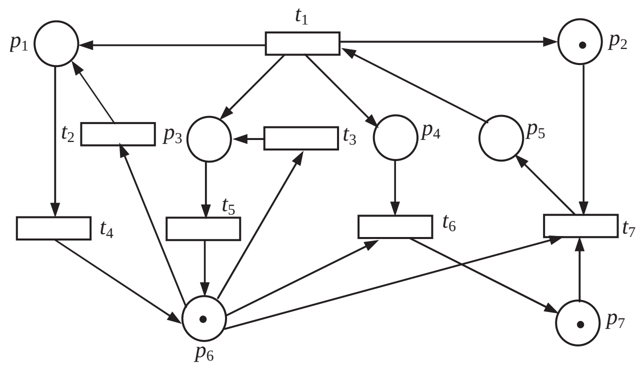

Fig. 10. A live and bounded OAC net (N, M_0) with a non-live and unbounded D-induced subnet.

$t_3, t_4, t_5, t_6, t_7\}, F_D), (0\ 0\ 0\ 1))$ induced by the minimal siphon $D = \{p_1, p_3, p_5, p_6\}$ is neither live nor bounded, because firing the sink transition t_6 within N_D will lose all tokens and firing the sequence $t_7 t_1 t_5$ infinitely many times will make p_1 become unbounded. Hence, Theorem 5.2 is not applicable. On the other hand, N is covered by two maximal siphons $\{p_1, p_3, p_4, p_5, p_6, p_7\}$ and $\{p_2, p_4, p_5, p_7\}$ whose induced subnets are live and bounded. It follows from Theorem 5.3 that (N, M_0) is live and bounded. However, (N, M) is not live with respect to the initial marking $M = (0\ 1\ 0\ 1\ 0\ 1\ 1) \geqslant M_0$.

Example 5.2. This example shows that Theorem 5.3 is not applicable as a necessary condition. The OAC net (N, M_0) of Example 4.3 in Fig. 6 is live and bounded for $M_0 = (0, 0, 1, 0, 0, 0)$. N is covered by two maximal siphons $D_1 = \{p_1, p_3, p_6\}$ and $D_2 = \{p_2, p_3, p_4, p_5, p_6\}$. But, the induced subnet $(D_2, (D_2)^\bullet, F_{D2}) = (\{p_2, p_3, p_4, p_5, p_6\}, \{t_1, t_2, t_3, t_4, t_5\}, F_{D2})$ is not bounded.

Example 5.3. Consider the HAC net (N, M_0) of Example 4.1 in Fig. 3 for $M_0 = (1, 0, 2, 0)$. N is covered by two minimal siphons $D_1 = \{p_1, p_2\}$ and $D_2 = \{p_3, p_4\}$. Their induced subnets $(D_1, (D_1)^\bullet, F_{D1}) = (\{p_1, p_2\}, \{t_1, t_2\}, F_{D1})$ and $(D_2, (D_2)^\bullet, F_{D2}) = (\{p_3, p_4\}, \{t_2, t_3, t_4\}, F_{D2})$ are live and bounded. According to Theorem 5.2, the net (N, M) is live and bounded for any $M \geqslant M_0$.

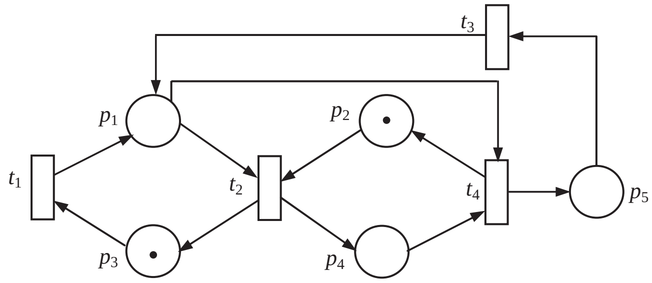

Fig. 11. A live, bounded but non-reversible ST-OAC net.

It is well known that a live and bounded OFC net (N, M_0) is reversible if M_0 marks every trap of N. However, as shown in Example 5.4 below, this characterization cannot be extended even to ST-OAC nets.

Example 5.4. The Petri net (N, M_0) in Fig. 11 is a live and bounded ST-OAC net. M_0 marks all minimal traps $\{p_1, p_3, p_5\}$ and $\{p_2, p_4\}$. However, after t_1 is fired, the initial marking cannot be reached again. Hence, this initial marking is not reversible.

6. Application to the resource-sharing problem

Based on the results obtained in the previous sections, this section first presents a property-preserving method for merging some places of an AC net. The method is then applied to solve some resource-sharing problems in system design.

6.1. Preservation of properties under merge of places

This subsection presents a method for merging some subsets of places of an OAC net. Conditions under which the merge will preserve the ST-property, asymmetric-free-choice-ness, liveness, boundedness and reversibility of the original net will be proposed.

MERGE-PLACE. Suppose (N, M_0) is a net, where $N=(P_0 \cup Q_1 \cup \cdots \cup Q_k, T, F)$ satisfies the condition: For $i, j = 1, 2, \ldots, k$, where $i \neq j, P_0 \cap Q_i = \emptyset$, $Q_i \cap Q_j = \emptyset$, and $\forall p, q \in Q_i : ({}^{\bullet}p \cap {}^{\bullet}q) = \emptyset$ and $(p^{\bullet} \cap q^{\bullet}) = \emptyset$. Let (N', M_0') be obtained from (N, M_0) by merging the places of each Q_i into q_i and creating the initial marking M_0' as follows:

- $N' = (P_0 \cup Q_0,\ T', F')$, where $Q_0 = \{q_1, q_2, \ldots, q_k\}$, $T' = T$, and F' is obtained from F by replacing every arc of the form (t, p) or (p, t), where $p \in Q_i$, by (t, q_i) or (q_i, t), respectively.
- M_0' is obtained by one of the following two rules:

  Rule 1:

$$M_0'(p) = \begin{cases} M_0(p), & p \in P_0, \\ \max_{q \in Q_i}\{M_0(q)\}, & p = q_i \in Q_0. \end{cases}$$

Rule 2 (This rule can be adopted only if $M_0(q)=M_0(q')\forall q,q'\in Q_i$ for $i=1,2,\ldots,k$):

$$M_0'(p)=\begin{cases} M_0(p), & p\in P_0,\\ M_0(q), & p=q\in Q_0.\end{cases}$$

In general, the net (N',M_0') obtained by MERGE-PLACE may not preserve some of the properties of (N,M_0). As Theorem 6.1 will show, however, for certain classes of AC nets, liveness, boundedness and reversibility will be preserved if some conditions are imposed on (N,M_0). To prove this theorem, we need the following lemma.

Lemma 6.1 (Preservation of siphons and traps under MERGE-PLACE for general Petri nets)**.** *Suppose* $N'=(P_0\cup Q_0,T',F')$ *is obtained from* $N=(P_0\cup Q_1\cup\cdots\cup Q_k,T,F)$ *by MERGE-PLACE. For any* $D'\subseteq P_0\cup Q_0$, *let* $D=\cup Q_i\mid \exists q_i\in D'\cap Q_0\cup(D'\cap P_0)$. *Then*, D' *is a siphon* (*resp*., *trap*) *of* N' *iff* D *is a siphon* (*resp*., *trap*) *of* N.

Proof. According to MERGE-PLACE, $D'\cap P_0=D\cap P_0$ and $\forall q_i\in Q_0$: $q_i^\bullet$ in $N'=Q_i^\bullet$ in N and ${}^\bullet q_i$ in $N'={}^\bullet Q_i$ in N.
($\Rightarrow$); $\forall t\in{}^\bullet D$ (resp., $t\in D^\bullet$) in N, consider two cases: *Case* 1 ($t\in{}^\bullet(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})$) (*resp*., $t\in(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})^\bullet$): Then, $t\in{}^\bullet(D'\cap Q_0)\subseteq{}^\bullet D'\subseteq D'^\bullet$ (resp., $t\in(D'\cap Q_0)^\bullet\subseteq D'^\bullet\subseteq{}^\bullet D'$). Hence, $t\in(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})^\bullet\cup(D'\cap P_0)^\bullet=D^\bullet$ (resp., $t\in{}^\bullet(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})\cup{}^\bullet(D'\cap P_0)={}^\bullet D$) in N. *Case* 2 ($t\in{}^\bullet(D'\cap P_0)$) (*resp*., $t\in(D'\cap P_0)^\bullet$): Then, $t\in{}^\bullet D'\subseteq D'^\bullet=(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})^\bullet\cup(D'\cap P_0)^\bullet=D^\bullet$ (resp., $t\in D'^\bullet\subseteq{}^\bullet D'={}^\bullet(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})\cup{}^\bullet(D'\cap P_0)={}^\bullet D$) in N. Hence, D is also a siphon (resp., trap) of N.
($\Leftarrow$): $\forall t\in{}^\bullet D'$ (resp., $t\in D'^\bullet$) in N'. *Case* 1($t\in{}^\bullet(D'\cap Q_0)$ (*resp*., $(D'\cap Q_0)^\bullet$): Then, $t\in{}^\bullet(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})\subseteq{}^\bullet D\subseteq D^\bullet$ (resp., $t\in(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})^\bullet\subseteq D^\bullet\subseteq{}^\bullet D$). Hence, $t\in(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})^\bullet\cup(D'\cap P_0)^\bullet=((D'\cap Q_0)\cup(D'\cap P_0))^\bullet=D'^\bullet$ (resp., $t\in{}^\bullet(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\}\cup(D'\cap P_0))={}^\bullet((D'\cap Q_0)\cup(D'\cap P_0))={}^\bullet D'$) in N'. *Case* 2 ($t\in{}^\bullet(D'\cap P_0)$ (*resp*., $t\in(D'\cap P_0)^\bullet$): Then, $t\in{}^\bullet(D\cap P_0)\subseteq{}^\bullet D\subseteq D^\bullet=(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})^\bullet\cup(D'\cap P_0)^\bullet=((D'\cap Q_0)\cup(D'\cap P_0))^\bullet=D'^\bullet$ (resp., $t\in(D\cap P_0)^\bullet\subseteq D^\bullet\subseteq{}^\bullet D={}^\bullet(\cup\{Q_i\mid\exists q_i\in D'\cap Q_0\})\cup{}^\bullet(D'\cap P_0)={}^\bullet(D'\cap Q_0)\cup{}^\bullet(D'\cap P_0)={}^\bullet D'$). Hence, D' is also a siphon (resp., trap) of N'. □

It follows from Corollary 4.1 that a well-formed ST-OAC net has a positive P-invariant. Theorem 6.1 below states that, for a live and bounded ST-OAC net, MERGE-PLACE can preserve liveness and boundedness under some conditions.

Theorem 6.1. *Let* (N,M_0) *be a live, bounded and reversible ST-OAC net. Suppose the positive* P-*invariant* $\alpha=(a_1,a_2,\ldots,a_{|P0|},a_{|P0|+1},\ldots,a_{|P0|+|Q1|},\ldots,a_{|P0|+|Q1|+\cdots+|Qk|})$ *satisfies* $a_{|P0|}=\cdots=a_{|P0|+|Q1|},a_{|P0|+|Q1|+1}=\cdots=a_{|P0|+|Q1|+|Q2|},\ldots,a_{|P0|+|Q1|+\cdots+|Qk-1|+1}=\cdots=a_{|P0|+|Q1|+\cdots+|Qk|}$. *Then, the net* (N',M_0') *obtained from* (N,M_0) *by MERGE-PLACE is live, bounded and reversible if* N' *is also ST-OAC net.*

Proof. Let D' be any siphon of N' containing a trap S'. By Lemma 6.1, $S = \cup\{Q_i \mid \exists q_i \in S' \cap Q_0\} \cup (S' \cap P_0)$ is a trap of N. Since (N, M_0) is live and reversible, by Property 1.5, S is marked by M_0. Since $S' \cap P_0$ in $N' = S \cap P_0$ in N and $q_i \in S' \Leftrightarrow Q_i \subseteq S, S'$ is marked by M_0'. It follows from Property 1.2 that (N, M_0) is live.

The incidence matrices V and V' of N and N' have the following forms, respectively:

$$V = \begin{matrix} \\ P_0 \\ Q_1 \\ Q_2 \\ \cdots \\ Q_k \end{matrix} \begin{matrix} \begin{matrix} t_1 & t_2 & \cdots & t_m \end{matrix} \\ \begin{pmatrix} V_{01} & V_{02} & \cdots & V_{0m} \\ V_{11} & V_{12} & \cdots & V_{1m} \\ V_{21} & V_{22} & \cdots & V_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ V_{k1} & V_{k2} & \cdots & V_{km} \end{pmatrix} \end{matrix} \quad \text{and } V' = \begin{matrix} \\ P_0 \\ q_1 \\ q_2 \\ \cdots \\ q_k \end{matrix} \begin{matrix} \begin{matrix} t_1 & t_2 & \cdots & t_m \end{matrix} \\ \begin{pmatrix} V_{01} & V_{02} & \cdots & V_{0m} \\ u_{11} & u_{12} & \cdots & u_{1m} \\ u_{21} & u_{22} & \cdots & u_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ u_{k1} & u_{k2} & \cdots & u_{km} \end{pmatrix} \end{matrix},$$

$$\text{where } V_{ij} = \begin{matrix} \\ q_{i1} \\ q_{i2} \\ \cdots \\ q_{t|Q_i|} \end{matrix} \begin{matrix} t_j \\ \begin{pmatrix} v_{i1,j} \\ v_{i2,j} \\ \cdots \\ v_{i|Qi|,j} \end{pmatrix} \end{matrix}, \quad i = 1, \ldots, k \text{ and } j = 1, \ldots, m.$$

According to the definition of MERGE-PLACE, we know that

$$u_{ij} = \sum_{l=1}^{|Q_i|} v_{il,j} \text{ for } i = 1, \ldots, k \text{ and } j = 1, \ldots, m.$$

Since $\alpha = (a_1, a_2, \ldots, a_{|P0|}, a_{|P0|+1}, \ldots, a_{|P0|+|Q1|}, \ldots, a_{|P0|+|Q1|+\cdots+|Qk|}) \geqslant 0$ is a positive of N, $\alpha V = 0$, where $a_{|P0|} = \cdots = a_{|P0|+|Q1|}, a_{|P0|+|Q1|+1} = \cdots = a_{|P0|+|Q1|+|Q2|}, \ldots, a_{|P0|+|Q1|+\cdots+|Qk-1|+1} = \cdots = a_{|P0|+|Q1|+\cdots+|Qk|}$. $\alpha V = 0$. Let $\alpha' = (a_1, a_2, \ldots, a_{|P0|}, a_{|Q1|}, a_{|Q2|}, \ldots, a_{|Qk|})$. Since $u_{ij} = \sum_{l=1}^{|Q_i|} v_{il,j}$ for $i = 1, \ldots, k$ and $j = 1, \ldots, m$, then $\alpha' V' = 0$, i.e., α' is also a positive P-invariant of N'. Hence, N' is structurally bounded. This means that (N', M_0') is bounded.

For any $M' \in R(N', M_0')$, since (N', M_0'), is live and bounded, there exists $M'' \in R(N', M')$ such that M'' is the marking obtained from $M \in R(N, M_0)$. Since (N, M_0) is reversible, $M_0 \in R(N, M)$. Hence, $M_0' \in R(N', M'') \subseteq R(N', M')$. □

As shown in Example 6.2 below, the ST-property plays an essential role in Theorem 6.1. Furthermore, in order to apply Theorem 6.1, one has to show that the created net N' is a pure OAC net and satisfies the ST-property. For complex nets, this may not be a simple task. Theorem 6.2 below presents a necessary and sufficient condition on N for the preservation of asymmetric-free-choice-ness and Theorem 6.3 presents a sufficient condition on N for the preservation of the ST-property.

Example 6.1. Theorem 6.1 may not be valid without the ST-property. For example, the net in Fig. 12 is a live, bounded and reversible OAC net, but, the OAC net N' obtained by merging q_1 and q_2 into q under Rule 2 is not live. Both N and N' do not satisfy the ST-property.

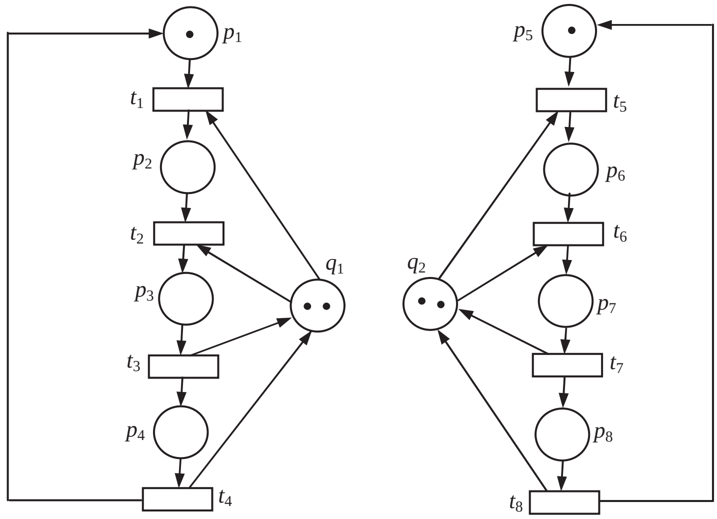

Fig. 12. A live, bounded and reversible net with shared places q_1 and q_2.

Theorem 6.2 (MERGE-PLACE preserves asymmetric-free-choice-ness). *Suppose* $N' = (P_0 \cup Q_0, T', F')$ *is obtained from an OAC net* $N = (P_0 \cup Q_1 \cup \cdots \cup Q_k, T, F)$ *by MERGE-PLACE. Then,* N' *is an OAC net if the following conditions hold in* N:
(1) $\forall p \in P_0 \forall q \in Q_1 \cup \cdots \cup Q_k$, *if* $p^\bullet \cap q^\bullet \neq \emptyset$ *then* $p^\bullet \subseteq q^\bullet$.
(2) *If* $Q_i^\bullet \cap Q_j^\bullet \neq \emptyset$, *then* $Q_i^\bullet \subseteq Q_j^\bullet$ *or* $Q_j^\bullet \subseteq Q_i^\bullet$.

Proof. In N', $\forall p, q \in P_0 \cup Q_0$, where $p^\bullet \cap q^\bullet \neq \emptyset$, consider three cases. *Case* 1 (*Both* $p, q \in P_0$): p and q are in P_0 in N and their pre-sets and post-sets remain unchanged in N'. Since N is an OAC net, $p^\bullet \subseteq q^\bullet$ or $q^\bullet \subseteq p^\bullet$ in N and, thus, also in N'. *Case* 2 ($p \in P_0$ *but* $q \in Q_0$): Suppose $q = q_i \in Q_i$ for some i, then $q^\bullet$ in $N' = Q_i^\bullet$ in N. By Condition (1), $p^\bullet \subseteq Q_i^\bullet$ in N. Hence, $p^\bullet \subseteq q^\bullet$ in N'. *Case* 3 (*Both* $p, q \in Q_0$): Since N is OAC, Condition (2) implies $p^\bullet \subseteq q^\bullet$ or $q^\bullet \subseteq p^\bullet$ in N'. $\square$

Theorem 6.3 (MERGE-PLACE preserves the ST-property for general Petri nets). *Suppose* $N' = (P_0 \cup Q_0, T', F')$ *is obtained from* $N = (P_0 \cup Q_1 \cup \cdots \cup Q_k, T, F)$ *by MERGE-PLACE. Then,* N' *satisfies the ST-property if the following two conditions hold in* N:
(1) N *satisfies the ST-property.*
(2) *For every siphon* D *of* N, *if* $Q_i \subseteq D$, *where* $i \in \{1, 2, \ldots, k\}$, *then* D *must contain a trap* S *such that either* $S \subseteq P_0$ *or* $Q_i \subseteq S$.

Proof. For an arbitrary siphon D' of N', let $D = \cup\{Q_i \mid \exists q_i \in D' \cap Q_0\} \cup (D' \cap P_0)$. By Lemma 6.1, D is a siphon of N. If $D' \cap Q_0 = \emptyset$, then $D \subseteq P_0$ and thus contains at least one trap S of N and S is also a trap of N'. If $q_i \in D'$, then $Q_i \subseteq D$. By Condition (1), D contains a trap S_i such that either $S_i \subseteq P_0$ or $Q_i \subseteq S$. Let S be the union of these traps. Then, S is a trap of N. Let $S' = \cup\{q_i | Q_i \subseteq S\} \cup (S \cap P_0)$. Since $S \cap P_0 = S' \cap P_0$, $S = \cup\{Q_i | \exists q_i \in S' \cap Q_0\} \cup (S' \cap P_0) \subseteq D$. Hence, $S' \subseteq D'$. By Lemma 6.1, S' is a trap of N'. Hence, N' satisfies ST-property. □

6.2. Application of MERGE-PLACE to the verification of resource sharing systems

This subsection applies results of Section 6.1 to solve some resource-sharing problems. Since the methodology is already implicitly included in the MERGE-PLACE process, it is adequate to just illustrate the method by an example.

Example 6.2. The live, bounded and reversible OAC net (N, M_0) of Fig. 13(a) is modified from [10], with $P_0 = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}, p_{11}, p_{12}, p_{13}\}$, $Q_1 = \{q_{11}, q_{12}\}$ and $Q_2 = \{q_{21}, q_{22}, q_{23}, q_{24}\}$.

The vector $\alpha = (2\,1\,1\,2\,1\,1\,2\,1\,1\,3\,1\,2\,1\,1\,1\,1\,1\,1\,1)$ corresponding to the order $(p_1\, p_2\, p_3\, p_4\, p_5\, p_6\, p_7\, p_8\, p_9\, p_{10}\, p_{11}\, p_{12}\, p_{13}\ q_{11}\ q_{21}\ q_{12}\ q_{22}\ q_{23}\ q_{24})$ of the places is a positive P-invariant of N, where $\alpha(q_{11}) = \alpha(q_{21}) = 1$ and $\alpha(q_{12}) = \alpha(q_{22}) = \alpha(q_{23}) = \alpha(q_{24}) = 1$. The net N' in Fig. 13(b) is obtained from N by merging Q_1 into q_1 and Q_2 into q_2 under Rule 2. Obviously, N satisfies Condition (1) of Theorem 6.2. Since $Q_1^\bullet = \{t_1, t_{11}\}$ and $Q_2^\bullet = \{t_1, t_4, t_7, t_{11}, t_{13}\} \supseteq Q_1^\bullet$, Condition (2) of Theorem 6.2 is also satisfied. Hence, N' is an OAC net. Since all the minimal siphons $\{p_1, q_{11}\}$, $\{p_1, p_2, q_{21}\}$, $\{p_3, p_4, p_5, p_6, p_7\}$, $\{p_4, q_{22}\}$, $\{p_7, q_{23}\}$, $\{p_8, p_9\}$, $\{p_{10}, q_{12}\}$, $\{p_{10}, p_{11}, p_{12}, p_{13}\}$ and $\{p_{10}, p_{12}, q_{24}\}$ of N are also traps, Condition (1) of Theorem 6.3 is satisfied. It is easy to verify that N also satisfies Condition (2) of Theorem 6.3. For example, the siphon $D = \{p_1, q_{11}, p_{10}, q_{12}\}$ that contains Q_1 is itself a trap containing Q_1. Hence, N' satisfies the ST-Property. By Theorem 6.1, (N', M_0') is live, bounded and reversible.

Discussion on our approach for verifying a resource-sharing system. Our method for verification is based on merging the places representing the resources and aims at preserving the desirable properties under the merge. It provides the conditions on the original net for preserving liveness, boundedness and reversibility, the three important properties for practical applications. It also has the following advantages:

a. Our method preserves siphons and traps. As a consequence, when checking Condition (2) of Theorem 6.3, one has to check only those minimal siphons of N that contain at least one Q_i entirely. This greatly simplifies our verification process. (In Chu's method [7], one has to show that *every* minimal siphon contains a trap.)
b. Our method preserves asymmetric-free-choice-ness and ST-property. This simplifies the iterative approach for place merging. If one starts with an ST-OAC net, the net stays being ST-OAC after each merge. Note that, as illustrated in Example 6.1, preservation of the ST-property is an important feature of our method.
c. Our method is based on AC nets whereas most of the methods reported in the literature are based on state machines or marked graphs. It covers a wider scope of application.
d. Condition (1) of Theorem 6.2 is satisfied if the net is a state machine after eliminating the shared places. This includes, for example, Zhou's model [31] where the net is a state machine before inserting the mutual exclusions.

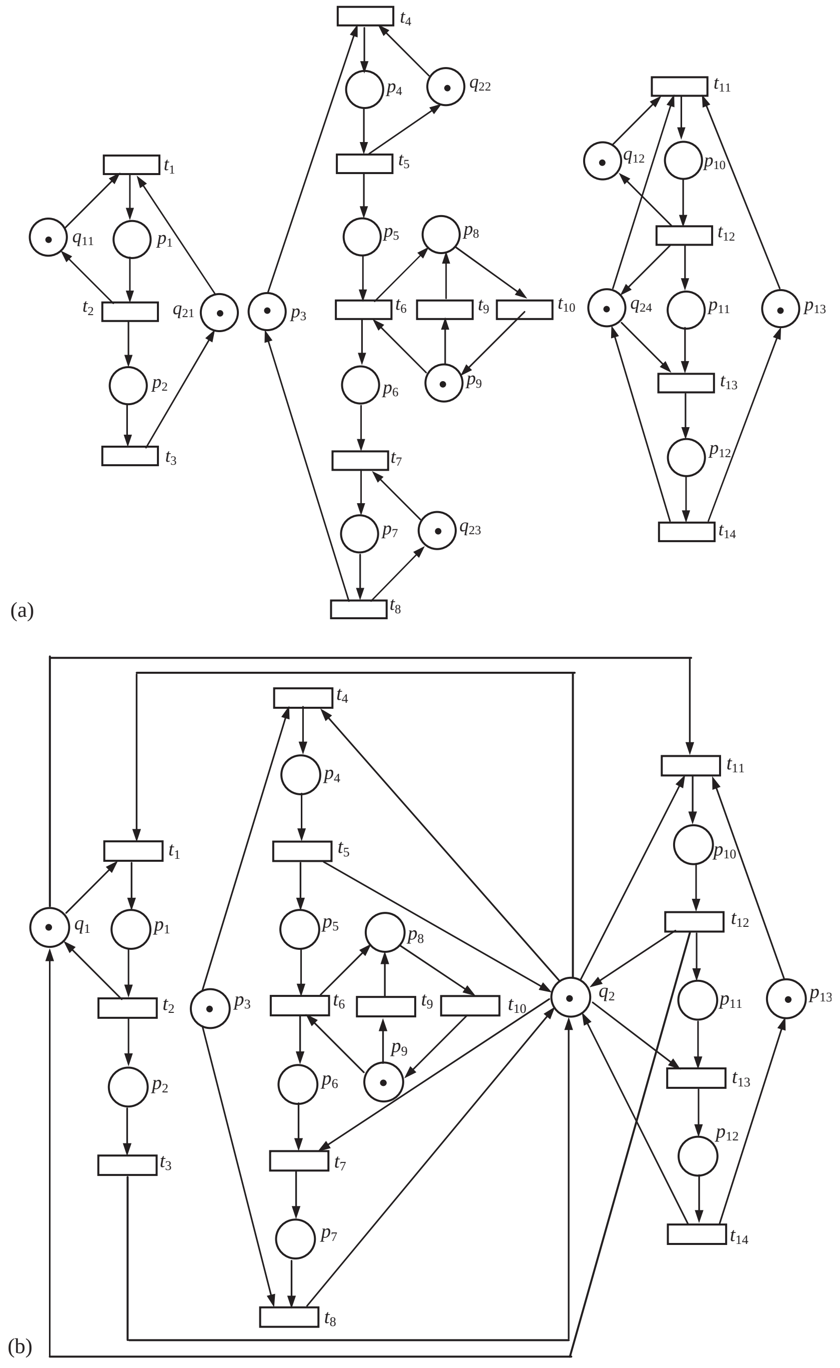

Fig. 13. (a) A live, bounded and reversible of ST-OAC net with two sets of shared places $\{q_{11}, q_{12}\}$ and $\{q_{21}, q_{22}, q_{23}, q_{24}\}$. (b) The live, bounded and reversible ST-OAC net obtained from (a) by MERGE-PLACE under Rule 2.

Table 5
A Classification (according to net types) of characterizations obtained in this paper

Type of net	Characterization or statement ($\Rightarrow$ means 'implies')	Ref.
OAC	Well-formed $\Rightarrow$ every minimal siphon either has no trap or is the only trap within itself	A8
	Well-formed does not $\Rightarrow$ structurally unbounded	A9a
ST-OAC	Live and bounded $\Rightarrow$ monotonically live	A1a
	Well-formed $\Rightarrow$ coverable by S-components	A6
	Not always coverable by T-components	A7
	Well-formed $\Leftrightarrow$ structurally bounded	A9b
	Live and bounded for M_0 does not $\Rightarrow$ reversibility even if M_0 marks every trap	A11
Well-formed	Live and bounded for $M_0 \Leftrightarrow$ every minimal siphon is marked by M_0	A10
ST-OAC	The RC-property is not always satisfied	A9a
HFC	Live $\Rightarrow$ monotonically live	A1b
HAC	A characterization for the non-liveness of individual transitions.	A2
	Subnet induced by every minimal siphon is live $\Leftrightarrow$ monotonically live	A3
	Subnet induced by every maximal siphon is live $\Rightarrow$ live	A4
	Well-formed $\Rightarrow$ coverable by minimal siphons	A5
	Live and bounded for any $M \geqslant M_0 \Leftrightarrow$ 'coverable by minimal siphons' and 'the subnet induced by every minimal siphon is live and bounded for M_0'	A12
	Live and bounded for $M_0 \Leftarrow$ 'coverable by maximal siphons' and 'the subnet induced by every maximal siphon is live and bounded for M_0'	A13

e. Condition (2) of Theorem 6.2 allows the post-sets of two different sets of resource places to intersect. This is more relaxed than most of the methods reported in the literature.

7. Summary and conclusion

This paper studies AC nets with two objectives, deriving characterizations for AC nets and applying these characterizations to solve some resource sharing problems. For the first objective, Table 5 summarizes our major results according to the type of the net. The main properties involved in the characterizations include: liveness, liveness monotonicity, coverability, well-formedness, and 'live and bounded with respect to a marking'. It is found that the ST-property plays an important role. For the second objective, this paper shows that, with some additional constraints, the properties asymmetric-free-choice-ness, liveness, boundedness and reversibility property are preserved after merging some sets of places for ST-OAC nets. As a consequence, this

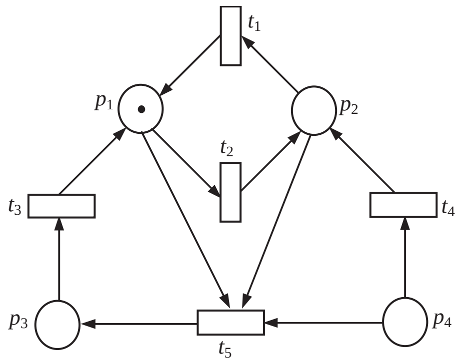

Fig. 14. A net showing that Property 1.2 cannot be extended to TAC nets.

result can be applied nicely to solve some of the resource-sharing problems in software engineering and manufacturing engineering. In comparison to most of the existing methods which are applied to state machines or marked graphs, our method is applicable to AC nets.

A main feature of this paper is the use of many examples for analyzing the different results and showing the limitation on further extension of these results to other classes of Petri nets.

In the literature, there are two kinds of asymmetric choice nets [2]: *PAC nets* defined in terms of places as in Section 1.2 and *TAC nets* defined in terms of transitions as follows: A net N is said to be a TAC net iff $\forall t_1,\ t_2 \in T : {}^\bullet t_1 \cap {}^\bullet t_2 \neq \emptyset \Rightarrow {}^\bullet t_1 \subseteq {}^\bullet t_2$ or ${}^\bullet t_2 \subseteq {}^\bullet t_1$. These are two different classes of Petri nets. All the results derived in this paper are for PAC nets. However, as illustrated in the following example, many similar results are not valid for TAC nets.

Example 7.1. This example shows that the characterization stated in Property 1.2 cannot be extended even to ordinary TAC nets. The net N in Fig. 14 is an ordinary TAC net because all weights are 1 and $({}^\bullet t_1 \cap {}^\bullet t_5 \neq \emptyset \Rightarrow {}^\bullet t_1 \subseteq {}^\bullet t_5)$ and $({}^\bullet t_2 \cap {}^\bullet t_5 \neq \emptyset \Rightarrow {}^\bullet t_2 \subseteq {}^\bullet t_5)$. Also, its only one siphon $\{p_1, p_2, p_3, p_4\}$ is a marked trap itself. However, the Petri net (N, M_0), where $M_0 = (1\ 0\ 0\ 0)$, is not live because, t_3, t_4, and t_5 can never be enabled. Hence, the net is not structurally bounded.

It needs further research to determine how to extend the results obtained in this paper for PAC nets to TAC nets and other more general types of nets. It is also interesting to compare the conclusion shown in Example 7.1 with that obtained in [2].

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